Public Economics (ECON 131)
Section #8: Public Goods

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Contents

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1 Key Concepts

- What are the definitions of good **rivalry** and **excludability**?
  - Are public goods non-rival and/or non-excludable?
- How do you achieve the **Social Demand Curve** (or Social Marginal Benefit Curve) from individual demand curves?
- What does the **Samuelson Rule** say?
- Why is there a **Free Rider Problem** with public goods?
- Why is there **Private Underprovision** of public goods?
  - What is the link with the free rider problem?

2 Practise Problems

2.1 Gruber, Ch.7, Q.13

The town of Springfield has two residents: Homer and Bart. The town currently funds its fire department solely from the individual contributions of these residents. Each of the two residents has a utility function over private goods (X_i) and total firefighters (M) of the form $U_i = 4 \cdot log(X_i) + 2 \cdot log(M)$, where $i = B, H$. The total provision of firefighters hired, M, is the sum of the number hired by each of the two persons: $M = M_H + M_B$. Homer and Bart both have income of \$100, and the price of both the private good and a firefighter is \$1. Thus, they are limited to providing between 0 and 100 firefighters.

(a) *How many firefighters are hired if the government does not intervene? How many are paid for by Homer? By Bart?*

Solution:

If each resident optimizes his own function, he will choose the number of firefighters that maximizes his own utility, taking into consideration the contribution by the other resident.

Private consumption, X_{Bart}, can be rewritten as $100 - M_{Bart}$ because all income not spent on firemen (M) can be spent on private goods.

The public good enjoyed by Bart can be rewritten as $M_{Bart} + M_{Homer}$ because public goods provided by either one are consumed by both.

Therefore, Bart's utility function can be rewritten as

$$U_{Bart} = 4 \cdot log(100 - M_{Bart}) + 2 \cdot log(M_{Bart} + M_{Homer}).$$

Set $\partial U / \partial M_{Bart}$ equal to zero:

$$-4/(100 - M_{Bart}) + 2/(M_{Bart} + M_{Homer}) = 0$$
$$4/(100 - M_{Bart}) = 2/(M_{Bart} + M_{Homer})$$

Cross-multiply, $4(M_{Bart} + M_{Homer}) = 2(100 - M_{Bart})$, and expand: $4M_{Bart} + 4M_{Homer} = 200 - 2M_{Bart}$.

Solving for M_{Bart} yields $M_{Bart} = (200 - 4M_{Homer})/6$.

The same procedure yields $M_{Homer} = (200 - 4M_{Bart})/6$.

These are response functions. They allow each resident to calculate his optimal M as a function of the contribution to M made by the other resident. Because the other resident's M carries a negative sign, the more one resident contributes, the less the other will.

Solving these response functions simultaneously is greatly eased by the fact that they are symmetric. At the solution, then, $M_{Homer} = M_{Bart}$, and hence $3M_{Bart} = 100 - 2M_{Homer} = 100 - 2M_{Bart}$. So $M_{Bart} = M_{Homer} = 20$.

(b) *What is the socially optimal number of firefighters? If your answer differs from part (a), why?*

Solution:

Method 1 - MRS: The socially optimal number is determined by adding each resident's marginal rate of substitution (placing the marginal utility for the public good in the numerator and for the private good in the denominator) and setting the result equal to the price ratio (1 here because both goods have the same price). Because Homer and Bart have the same utility functions, they will have the same marginal rates of substitution.

Therefore, the socially optimal number of firefighters solves $MRS^{Bart}_{M,X} + MRS^{Homer}_{M,X} = 1$.

Computing the MRS for each resident: $MU_M/MU_X = (2/M)/(4/[100 - M_i])$, where $M = M_{Homer} + M_{Bart}$, and $M_i = M_{Bart} = M_{Homer}$. The social optimum is then the solution to $(2/M)/(4/[100 - M_{Bart}]) + (2/M)/(4/[100 - M_{Homer}]) = 1$ or $[100 - M_{Bart}]/2M + [100 - M_{Homer}]/2M = 1$ or $200 - (M_{Bart} + M_{Homer}) = 2M$ or $200 - M = 2M$.

Hence, M = 200/3, and the social optimum is between 66 and 67 firefighters.

Method 2 - Lagrangian: Another way of doing this problem is to maximize the Lagrangian. The social planner maximizes $U_{Bart} + U_{Homer}$ by choosing $\{X_H, X_B, M_B, M_H\}$ subject to the budget constraints $100 = X_B + M_B$ and $100 = X_H + M_H$. This is equivalent to maximizing the following Lagrangian:

$$\begin{aligned} max_{\{X_H,X_B,M_B,M_H\}} L = & [4log(X_B) + 2log(M_H + M_B)] + [4log(X_H) + 2log(M_H + M_B)] \\ & + \lambda_1(100 - X_B - M_B) + \lambda_2(100 - X_H - M_H) \end{aligned}$$

Which gives first-order conditions:

(a) $\frac{4}{X_B} - \lambda_1 = 0$

(b) $\frac{4}{M_H+M_B} - \lambda_1 = 0$

(c) $\frac{4}{X_H} - \lambda_2 = 0$

(d) $\frac{4}{M_H+M_B} - \lambda_2 = 0$

(e) $100 - X_B - M_B = 0$

(f) $100 - X_H - M_H = 0$

There are a bunch of constraints, but they simplify quickly. Notice from (b) and (d) that $\lambda_1 = \lambda_2 = \frac{4}{M_H+M_B}$. Then using $\lambda_1 = \lambda_2$, we know from (a) and (b) that $\frac{4}{X_B} = \frac{4}{X_H}$ or $X_B = X_H$ which I define as X_i. Then from (e) and (f) we know that $M_B = M_H = 100 - X_i$, and so $X_i = 100 - M_i$. Then, from (a) and things we have derived we know that $\frac{4}{M_H+M_B} = \frac{4}{X_i}$, or $2M_i = X_i = 100 - M_i$. Solving, we get $M_i = 33.3$ or $M = M_B + M_H = 33.3 + 33.3 = 66.7$.

Intuitively, in the computation in part (a), we set the marginal utility of the last firefighter to each resident equal to the marginal utility of consumption for that resident. In part (b), we set the sum of the marginal utilities of the last firefighter - the social marginal utility of the firefighter - equal to the marginal utility of consumption for either resident. Since the social marginal utility of firefighters exceeds the individual marginal utilities of that firefighter, society optimally hires more than individuals would if they were acting alone.

2.2 Gruber, Ch.7, Q.15

Consider an economy with three types of individuals, differing only with respect to their preferences for monuments. Individuals of the first type get a fixed benefit of 100 from the mere existence of monuments, whatever their number. Individuals of the second and third type get benefits according to $B_{II} = 200 + 30M - 1.5M^2$ *and* $B_{III} = 150 + 90M - 4.5M^2$*, where M denotes the number of monuments in the city. Assume that there are 50 people of each type. Monuments cost $3,600 each to build. How many monuments should be built?*

Solution:

- The marginal benefit for type I individuals is 0 (if $M > 0$).
- The marginal benefit for type II individuals is $30 - 3M$.
- The marginal benefit for type III individuals is $90 - 9M$.
- The marginal cost is $3,600.

Aggregating marginal benefits and setting them equal to marginal cost yields:

$$50(30 - 3M) + 50(90 - 9M) = \$3,600 \qquad \Rightarrow \qquad M = 4$$

2.3 Gruber, Ch.7, Q.12

Andrew, Beth, and Cathy live in Lindhville. Andrew's demand for bike paths, a public good, is given by $Q = 12 - 2P$*. Beth's demand is* $Q = 18 - P$*, and Cathy's is* $Q = 8 - P/3$*. The marginal cost of building a bike path is* $MC = 21$*. The town government decides to use the following procedure for deciding how many paths to build. It asks each resident how many paths they want, and it builds the largest number asked for by any resident. To pay for these paths, it then taxes Andrew, Beth, and Cathy the prices* a*,* b*, and* c *per path, respectively, where* $a + b + c = MC$*. (The residents know these tax rates before stating how many paths they want.)*

(a) *If the taxes are set so that each resident shares the cost evenly* ($a = b = c$)*, how many paths will get built?*

Solution: When taxes are set at $a = b = c = MC/3 = 7$, each resident faces an individual marginal cost of 7 per bike path. At this marginal cost, Andrew wants no bike paths, Beth wants 11, and Cathy wants 5.67. The government therefore builds 11 paths.

(b) *Show that the government can achieve the social optimum by setting the correct tax prices* a*,* b*, and* c*. What prices should it set?*

Solution: The social optimum can be computed by reexpressing the demand curves for the three residents as $P = 6 - Q/2$, $P = 18 - Q$, and $P = 24 - 3Q$, respectively, and summing them to get marginal social benefit $MSB = 48 - 4.5Q$. Setting $MSB = MC$ and solving for Q gives $Q = 6$. We need to tax prices so that nobody will want more than 6 units (and someone will want exactly 6 units). Looking at Andrew's inverted demand curve, we see that he will want exactly 6 units at $a = 3$ (since then $a = 6 - 6/2$). Beth will want exactly 6 units at $b = 12$. And at $c = 6$, Cathy will want exactly 6 units. Since $3 + 12 + 6 = 21$, these tax rates are just enough to cover MC, and the social optimum is achieved. Note that with this tax system in place, the three residents are unanimous in the number of bike paths they desire.