ECON 3003 Advanced Public Economics
Fall 2022 – Prof. Dario Tortarolo

Tutorial 2

Labor Income Tax and Transfer

Assume people have the same utility function over consumption and labor given by:

$$U(c, l) = c + \theta \ln(16 - l)$$

where c represents consumption, l represents hours of labor, and θ is a constant parameter that reflects an individual's distaste for labor hours. Also, ln( ) denotes the natural logarithm (this can also be denoted by log( )). Suppose the only income that individuals have is from labor income, and that they work at an hourly wage w which is taxed at rate t.

(a) Assuming the government imposes a tax on labor income at rate t, write down the budget constraint faced by the individual. (1 point)

$$c = wl - twl = (1 - t)wl$$

(b) Solve for the individual's optimal labor supply as a function of w, θ, and t. (2 points)

Plugging the budget constraint into the utility function we get

$$U = (1 - t)wl + \theta \ln(16 - l)$$

Taking the first order condition:

$$\frac{\partial U}{\partial l} = (1 - t)w - \frac{\theta}{16 - l} = 0$$

Next, we isolate the optimal labor supply as a function of w, θ, and t

$$l^*(w, t, \theta) = 16 - \frac{\theta}{(1 - t)w}$$

(c) For this part (c) only, assume the following:

- There are 100 individuals in this society with the utility function given above.
- All individuals have $\theta = 24$.

- There are 50 low-wage individuals and 50 high-wage individuals. The low-wage individuals earn wage $w = 4$, while the high-wage individuals earn wage $w = 8$.

Calculate how much revenue the government will raise if it imposes a flat 25% labor income tax ($t = 0.25$) on all 100 individuals. (2 points)

Government's revenue (Rev) is equal to the tax rate times the tax base: $t \times wl$. We previously solved for optimal labor supply (l^*). Using that information we get:

$$Rev = tw\left[16 - \frac{\theta}{(1-t)w}\right]$$

The government collects from 50 low-wage individuals ($w = 4$):

$$Rev = 50 \times (0.25)(4)\left[16 - \frac{24}{(1-.25)(4)}\right] = (50)(1)(8) = 400$$

The government collects from 50 high-wage individuals ($w = 8$):

$$Rev = 50 \times (0.25)(8)\left[16 - \frac{24}{(1-.25)(8)}\right] = (50)(2)(12) = 1,200$$

So the total tax revenue is:

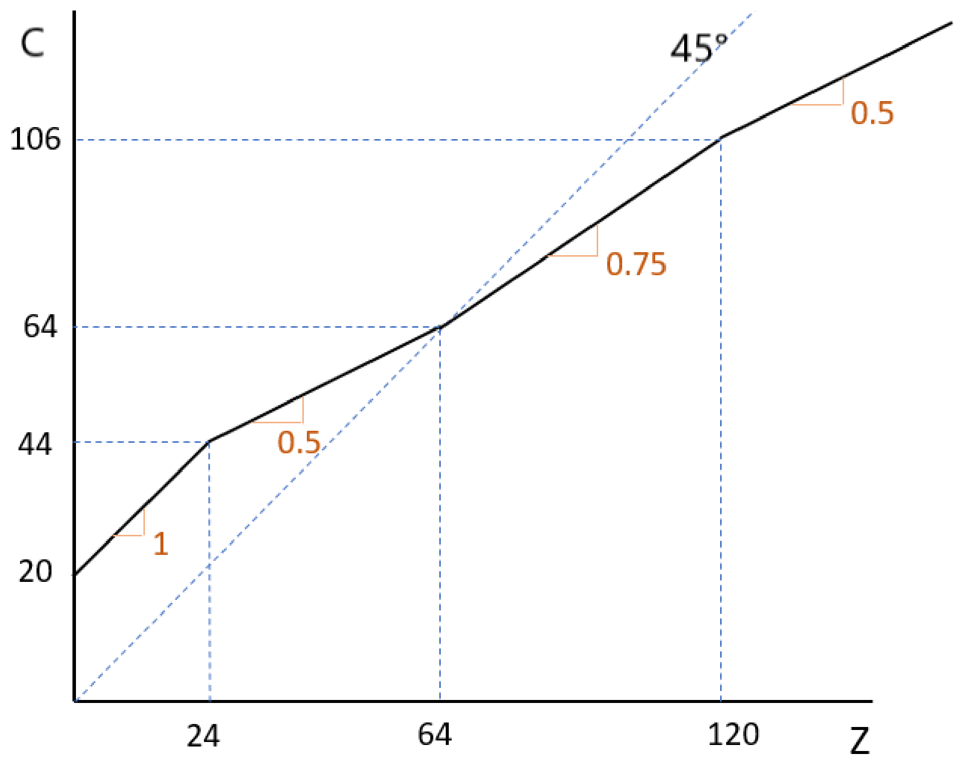
$$400 + 1,200 = 1,600$$

(d) The government creates a universal basic income (UBI) program to support individuals with low or zero labor income, where the UBI phases out after a certain threshold of income. The program is financed by imposing a labor tax on higher earners.

More specifically:

- The UBI program provides all individuals a lump-sum grant of £20.
- The grant does not phase out for the first £24 of labor income.
- After the first £24 of labor income, the grant is phased out at a 50% rate.
- After the grant phases out entirely, labor income from that point up to £120 is taxed at a 25% rate.
- After £120, any additional labor income is taxed at a 50% rate.

Graph the budget constraint (*Hint: there are four brackets*). Put pre-tax income ($Z = wl$) on the x-axis and after-tax income $c = Z - T(Z)$ on the y-axis. **Label the x and y values of each kink point, and the slope of each of the four segments of the graph.** (2 points)



Question: How did I come up with Z=64 (where the grant phases out entirely)?

The grant is 20 and the phase-out rate is 50%. Therefore, the grant will be fully exhausted when you earn an additional 40 (50% of 40 is 20), reaching 64.

How do we obtain 106 on the vertical axis? The question is, how much can this individual consume if they locate exactly at the third kink? You need to work it out with the piece-wise linear budget constraint. In the first bracket you can consume 44 (20 from grant + 24 labor income). In the second bracket, you earn income but you pay taxes. Hence, for the portion of income that you earn in that bracket, you can consume up to (64-24)*0.5 = 20. With the similar reasoning, in the third bracket you can consume up to (120-64)*0.75 = 42. Adding these three we get 44 + 20 + 42 = 106.

(e) For each of the four brackets in your budget constraint, indicate the sign (direction) of the substitution and income effects on an individual's choice of labor supply. Also indicate the sign (direction) of the total (combined) effect of the two, if it is possible to know for certain, or indicate with a question mark if the total effect is uncertain. (2 points)

	Bracket 1	Bracket 2	Bracket 3	Bracket 4
Substitution Effect:				
Income Effect:				
Total Effect:				

The general effects are:
SE: 0, $\downarrow$, $\downarrow$, $\downarrow$
IE: $\downarrow$, $\downarrow$, $\uparrow$, $\uparrow$
Tot: $\downarrow$, $\downarrow$, ?, ?

But with this specific utility function:
SE: 0, $\downarrow$, $\downarrow$, $\downarrow$
IE: 0, 0, 0, 0
Tot: 0, $\downarrow$, $\downarrow$, $\downarrow$

(f) For this part (f) only, consider and individual with $\theta = 24$ and $w = 8$. Calculate this individual's optimal choice of hours of labor l under the UBI program you graphed in part (d). (2 points)

We know the optimal labor supply is given by:

$$l^* = 16 - \frac{\theta}{(1-t)w} = 16 - \frac{24}{(1-t)8}$$

We can solve for each of the four brackets under the UBI program:

$$l^1 = 16 - \frac{24}{8} = 13 \quad (\text{so, } wl = 104)$$

$$l^2 = l^4 = 16 - \frac{24}{4} = 10 \quad (\text{so, } wl = 80)$$

$$l^3 = 16 - \frac{24}{6} = 12 \quad (\text{so, } wl = 96)$$

With $l^1 = 13$, $Z = 104$ is beyond the first segment; with $l^2 = 10$, $Z = 80$ is beyond the second segment; with $l^4 = 10$, $Z = 80$ is below the fourth segment; with $l^3 = 12$, $Z = 96$ is compatible with the third segment. Hence, the worker will choose $l^* = 12$ hours.

(g) A new administration is elected and they change the UBI policy:

- The grant is cut from £20 to £8.
- Workers are subsidized on the first £24 of labor earnings with a new credit that pays £0.50 per pound of pre-tax labor income. This new credit phases out at a 50% rate after the first £24 of income.

- All else stays the same.

Theoretically, how do you expect the change from the old UBI policy to this new policy to affect the extensive and intensive labor supply choices of individuals at and near the bottom of the income distribution? (2 points)
Keep answers succinct. No need for any calculations.

Extensive margin: The grant is reduced and the first bracket is steeper (higher after-tax wage), so we expect non-workers to enter the labor force.
Intensive margin: Since the first bracket is steeper (higher after-tax wage) and below the old bracket, we expect higher labor supply among workers in the first bracket.

And if you answered using the given utility function:
Extensive margin: Since there are no income effects, the grant reduction has no income effect. But the substitution effect of the higher after-tax wage on the phase-in portion will induce individuals with higher θ to enter the labor force and work.
Intensive margin: same as before.

(h) Next, the government raises the tax rate in the top bracket from 50% to 75%. If economists estimate that the elasticity of labor supply with respect to the after-tax rate for top earners in this society is equal to 1.5, show that this tax increase will not raise revenue. (1 point)
Hint: This is a self-contained question. You do not need to refer to any of your prior answers to answer this question.

Simple solution: Recall the 'Laffer curve' from lecture where we showed that the revenue-maximizing tax rate is $t^* = \frac{1}{1+e}$. Plugging in $e = 1.5$ yields: $t^* = \frac{1}{1+e} = 0.4$. Thus, any tax rate above 40% would be on the wrong side of the Laffer curve and decrease revenue.

Alternative solution: Revenue is given by: $Rev = t \times wl^*$. The wage rate w is exogenous and doesn't change. But when the government modifies the tax rate t, it triggers behavioral responses from individuals who change their labor supply. More formally:

We know $e = \frac{\Delta l/l}{\Delta(1-t)/(1-t)} = 1.5$, and we know $\Delta(1-t)/(1-t) = [(1-0.75)-(1-0.5)]/(1-0.5) = (0.25-0.5)/(0.5) = -0.25/0.5 = -0.5$. From the elasticity formula we have: $\Delta l/l = e \times \Delta(1-t)/(1-t) = 1.5 \times -0.5 = -75\%$.

So, labor supply will fall by 75% in the top bracket, while the tax rate is only going up by 50% (0.5 to 0.75). Thus, revenue will fall.