Theoretical Tools of Public Economics

Economía Pública: Impuestos
(Repaso)

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THEORETICAL AND EMPIRICAL TOOLS

Theoretical tools: The set of tools designed to understand the mechanics behind economic decision making.

Economists model individuals' choices using the concepts of utility function maximization subject to budget constraint

Narrow view of human behavior that works reasonably well for consumption choices but likely less well for work behavior

Empirical tools: The set of tools designed to analyze data and answer questions raised by theoretical analysis.

UTILITY MAPPING OF PREFERENCES

Utility function: A utility function is some mathematical function translating consumption into utility:

$$U = u(X_1, X_2, X_3, ...)$$

where X_1, X_2, X_3, and so on are the quantity of goods 1,2,3,... consumed by the individual

Example with two goods: $u(X_1, X_2) = \sqrt{X_1 \cdot X_2}$ with X_1 number of movies, X_2 number of music songs

Individual utility increases with the level of consumption of each good

PREFERENCES AND INDIFFERENCE CURVES

Indifference curve: A graphical representation of all bundles of goods that make an individual equally well off

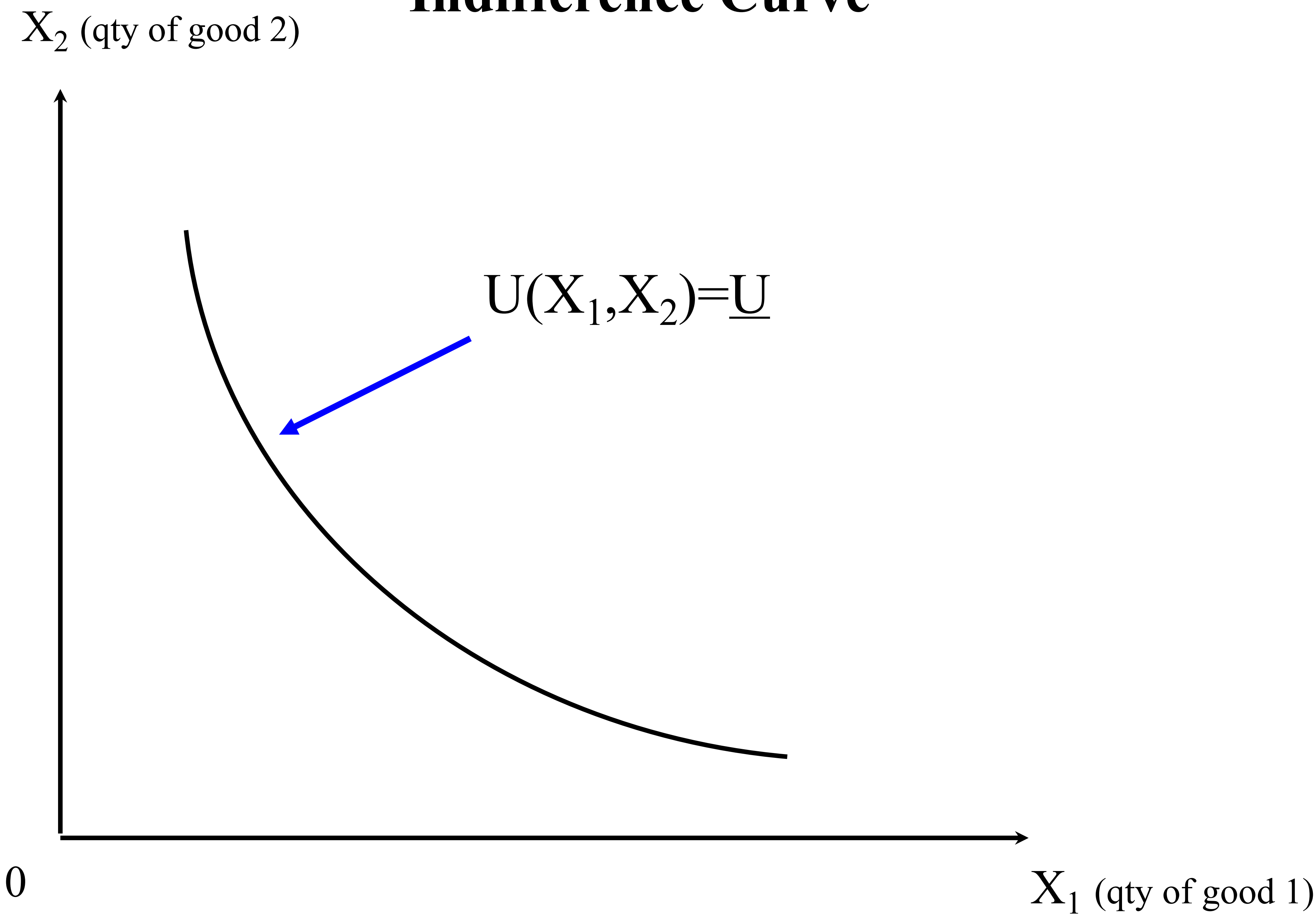
Mathematically, indifference curve giving utility level $\underline{U}$ is given by the set of bundles (X_1, X_2) such that $u(X_1, X_2) = \underline{U}$

Indifference curves have two essential properties, both of which follow naturally from the more-is-better assumption:

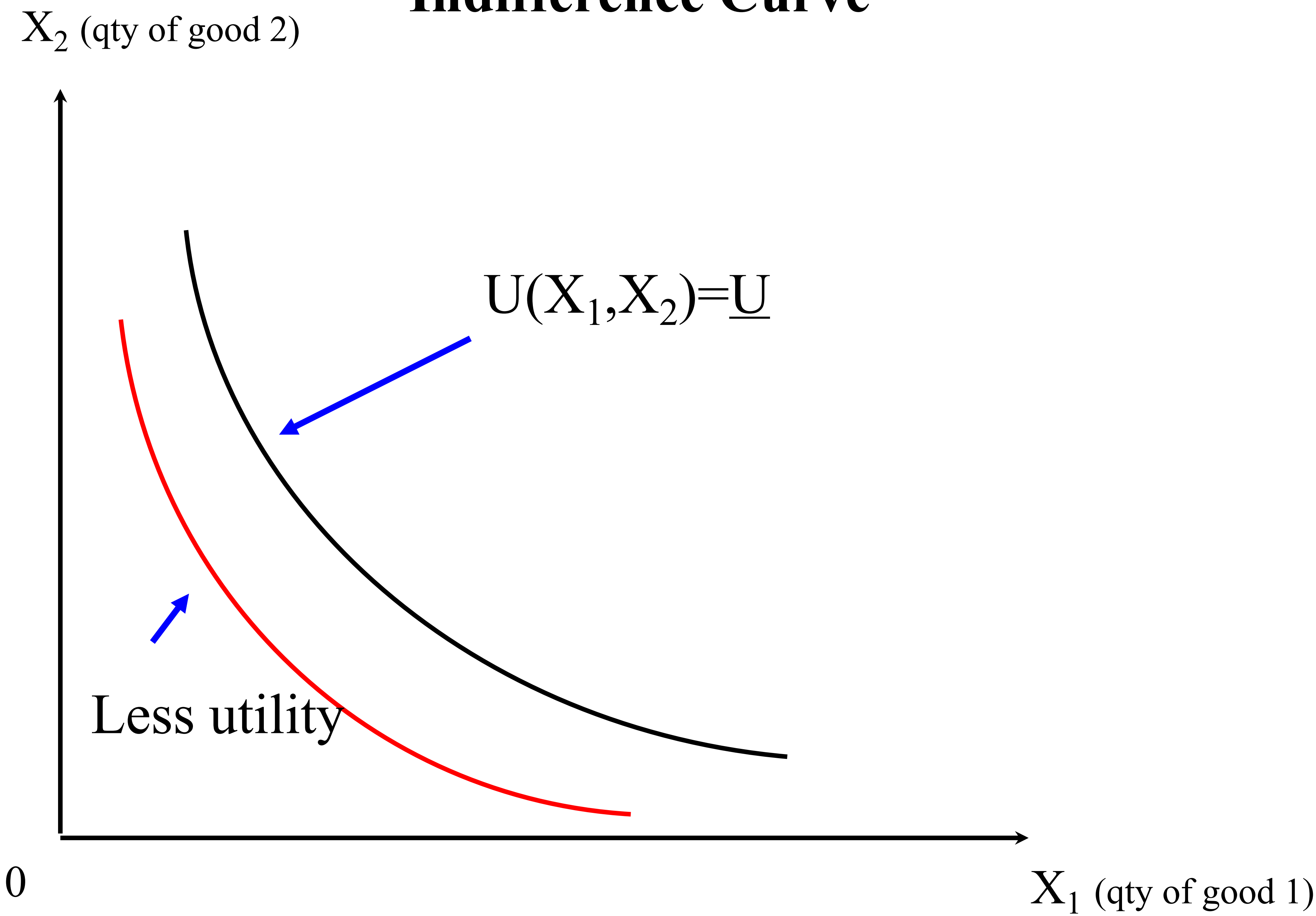
1. Consumers prefer higher indifference curves.

2. Indifference curves are always downward sloping.

Indifference Curve



Indifference Curve



MARGINAL UTILITY

Marginal utility: The additional increment to utility obtained by consuming an additional unit of a good:

Marginal utility of good 1 is defined as:

$$MU_1 = \frac{\partial u}{\partial X_1} \simeq \frac{u(X_1 + dX_1, X_2) - u(X_1, X_2)}{dX_1}$$

It is the derivative of utility with respect to X_1 keeping X_2 constant (called the partial derivative)

Example:

$$u(X_1, X_2) = \sqrt{X_1 \cdot X_2} \Rightarrow \frac{\partial u}{\partial X_1} = \frac{\sqrt{X_2}}{2\sqrt{X_1}}$$

This utility function described exhibits the important principle of **diminishing marginal utility**: $\partial u / \partial X_1$ decreases with X_1: the consumption of each additional unit of a good gives less extra utility than the consumption of the previous unit

MARGINAL RATE OF SUBSTITUTION

Marginal rate of substitution (MRS): The MRS is equal to (minus) the slope of the indifference curve, the rate at which the consumer will trade the good on the vertical axis for the good on the horizontal axis.

Marginal rate of substitution between good 1 and good 2 is:

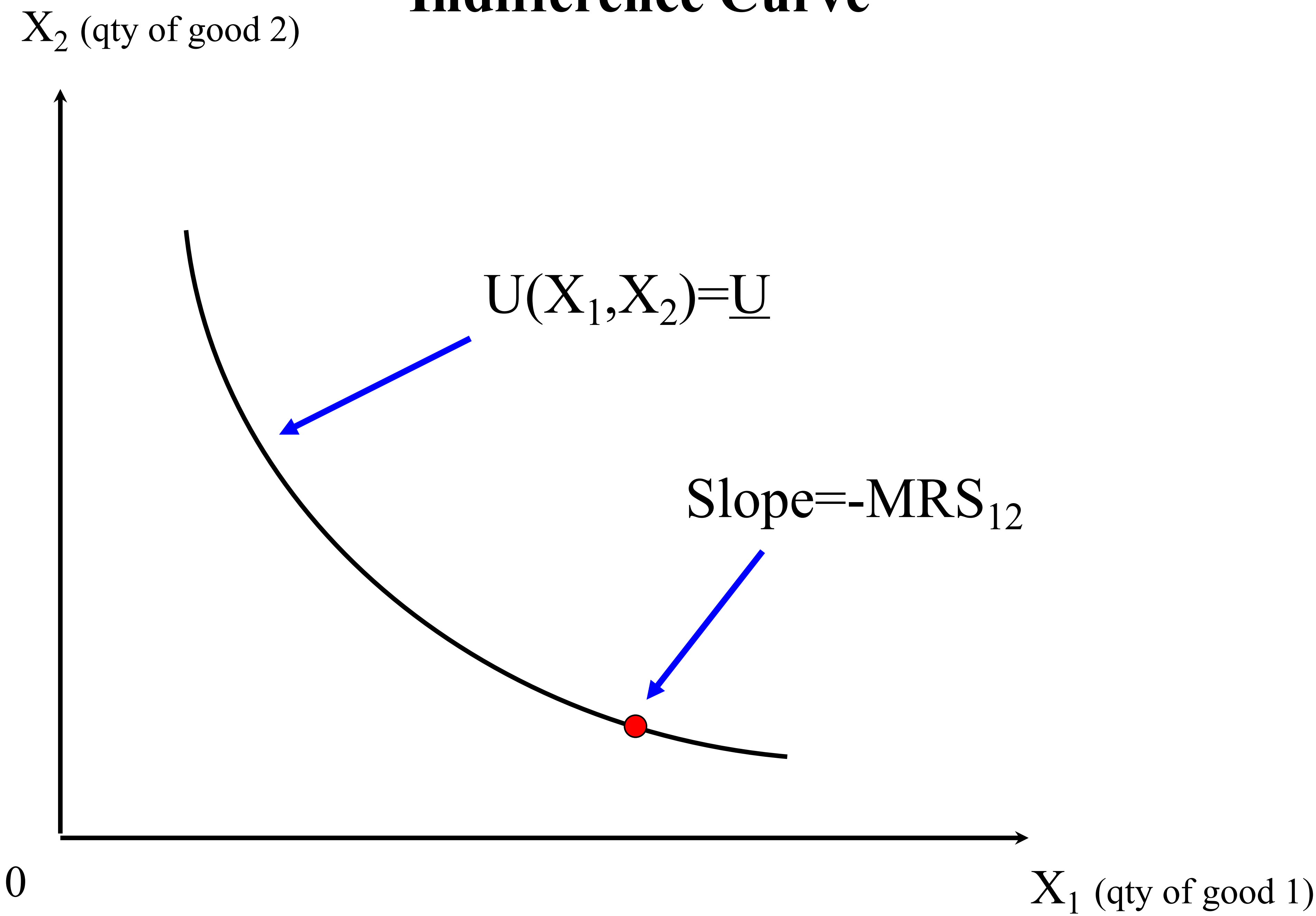
$$MRS_{1,2} = \frac{MU_1}{MU_2}$$

Individual is indifferent between 1 unit of good 1 and $MRS_{1,2}$ units of good 2.

Example:

$$u(X_1, X_2) = \sqrt{X_1 \cdot X_2} \Rightarrow MRS_{1,2} = \frac{X_2}{X_1}$$

Indifference Curve



BUDGET CONSTRAINT

Budget constraint: A mathematical representation of all the combinations of goods an individual can afford to buy if she spends her entire income.

$$p_1 X_1 + p_2 X_2 = Y$$

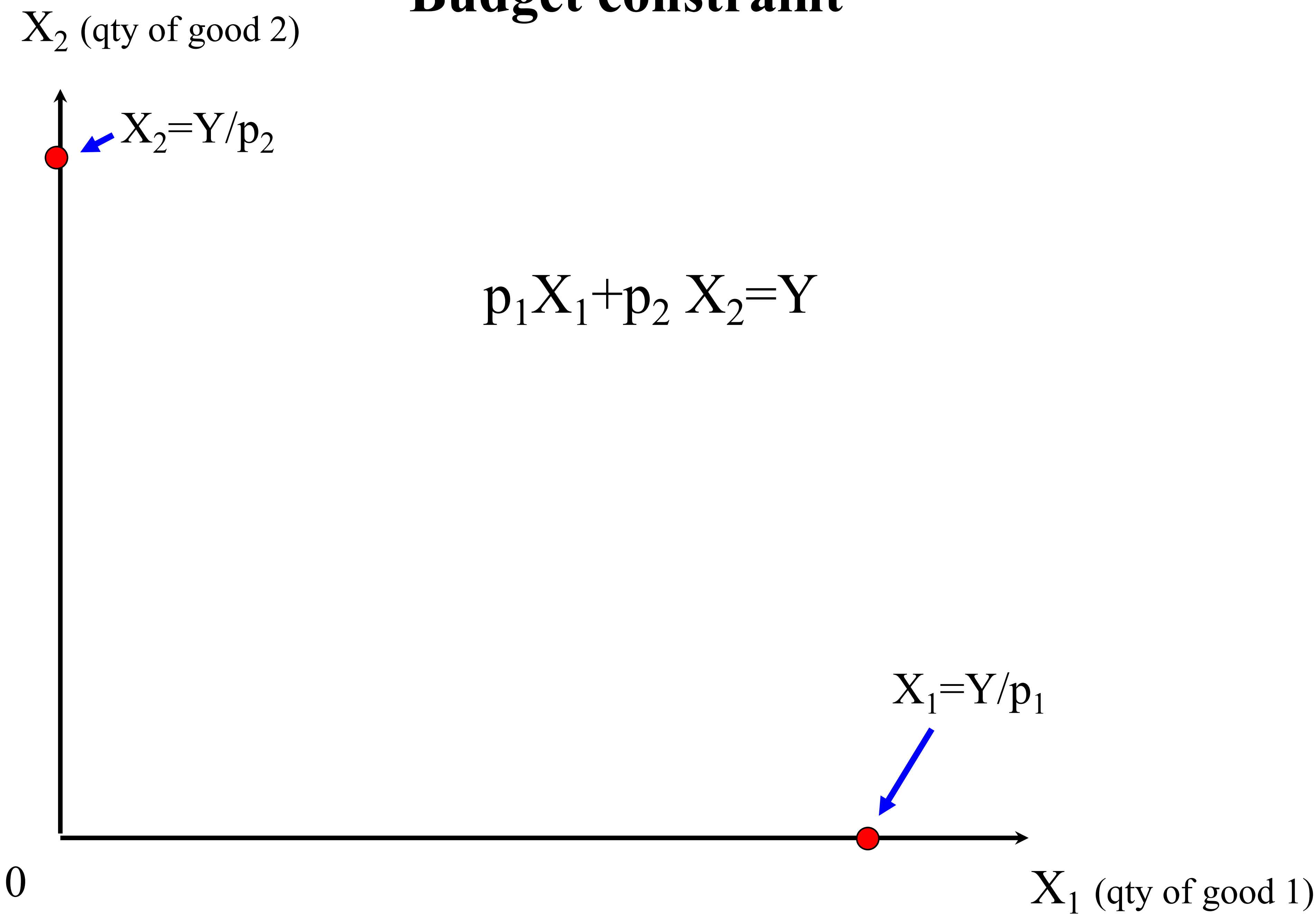
with p_i price of good i, and Y disposable income.

Budget constraint defines a linear set of bundles the consumer can purchase with its disposable income Y

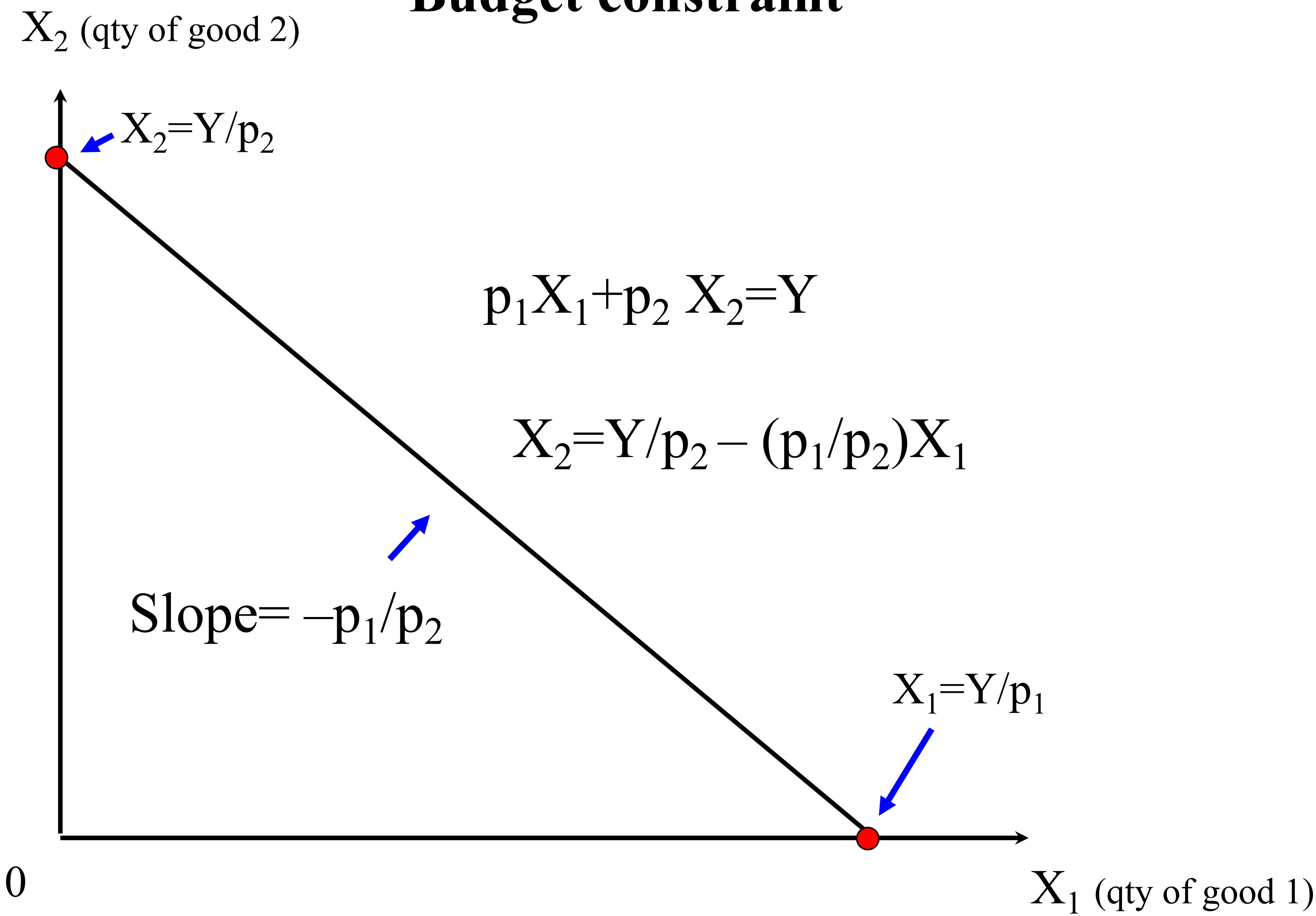
$$X_2 = \frac{Y}{p_2} - \frac{p_1}{p_2} X_1$$

The slope of the budget constraint is $-p_1/p_2$

Budget constraint



Budget constraint



UTILITY MAXIMIZATION

Individual maximizes utility subject to budget constraint:

$$\max_{X_1,X_2} u(X_1, X_2) \quad \text{subject to} \quad p_1X_1 + p_2X_2 = Y$$

$$\textbf{Solution:} \quad MRS_{1,2} = \frac{p_1}{p_2}$$

Proof: Budget implies that $X_2 = (Y - p_1X_1)/p_2$

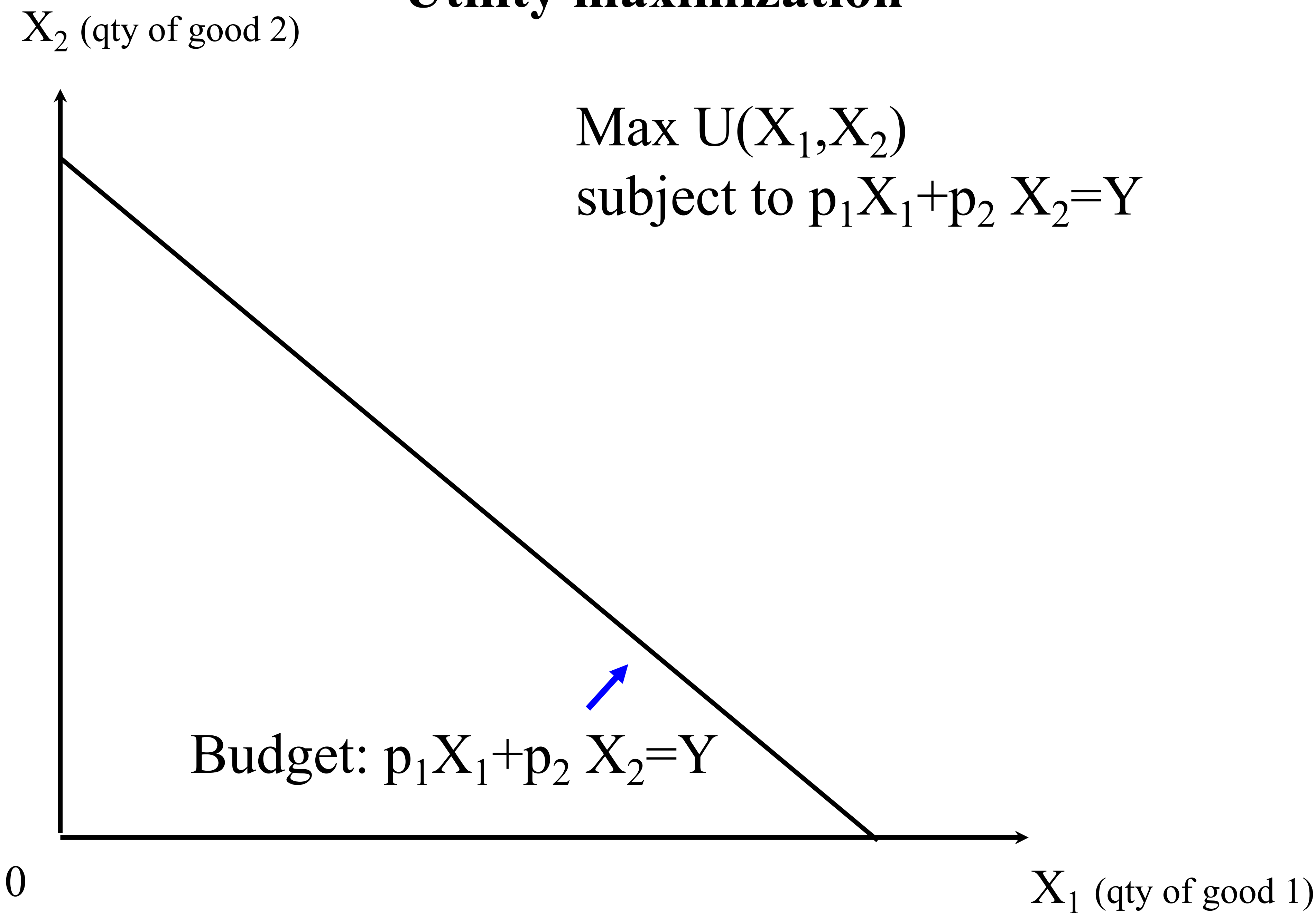
Individual chooses X_1 to maximize $u(X_1, (Y - p_1X_1)/p_2)$

The first order condition (FOC) is:

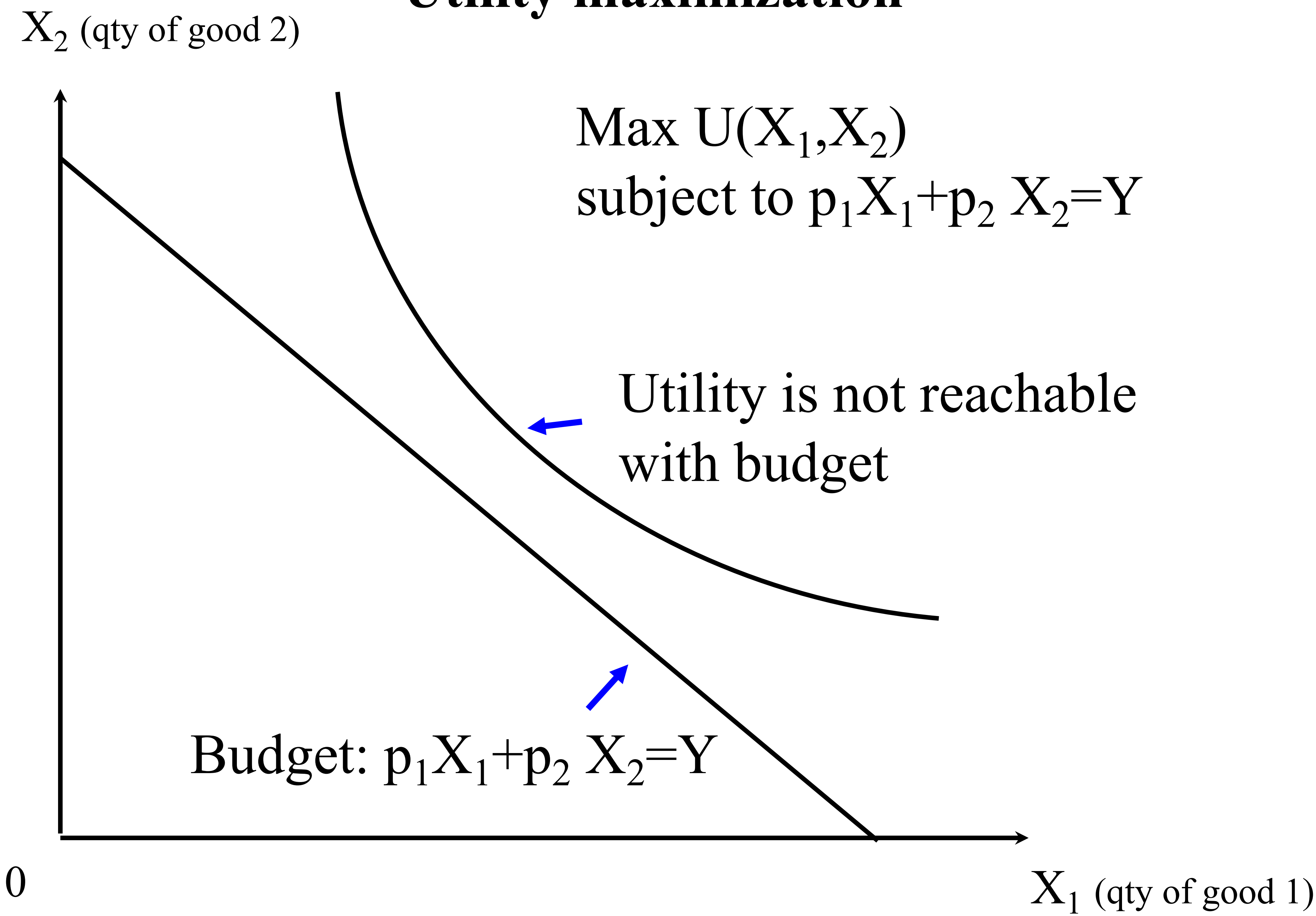
$$\frac{\partial u}{\partial X_1} - \frac{p_1}{p_2} \cdot \frac{\partial u}{\partial X_2} = 0.$$

At the optimal choice, the individual is indifferent between buying 1 extra unit of good 1 for \$ p_1 and buying p_1/p_2 extra units of good 2 (also for \$ p_1).

Utility maximization

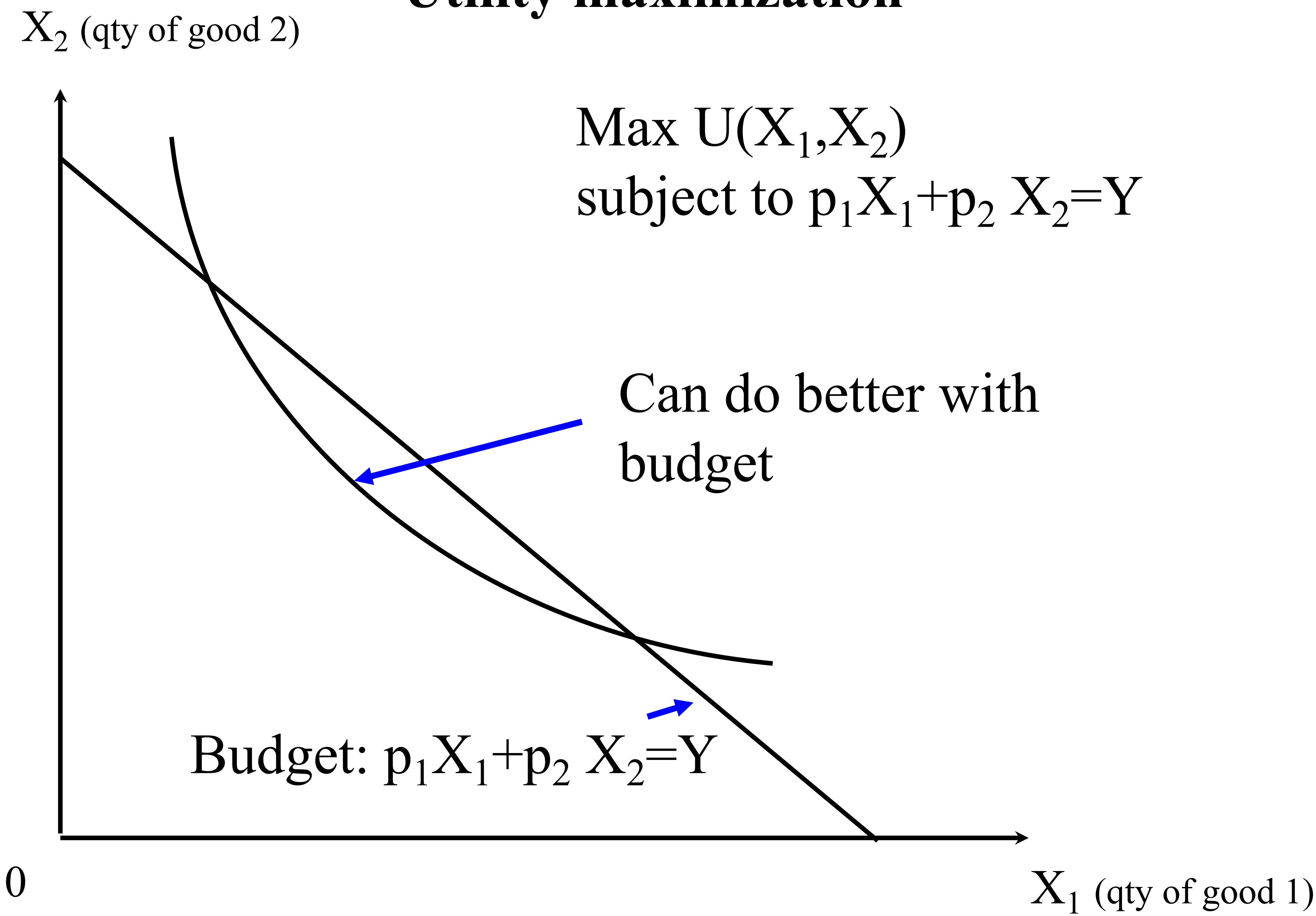


Utility maximization

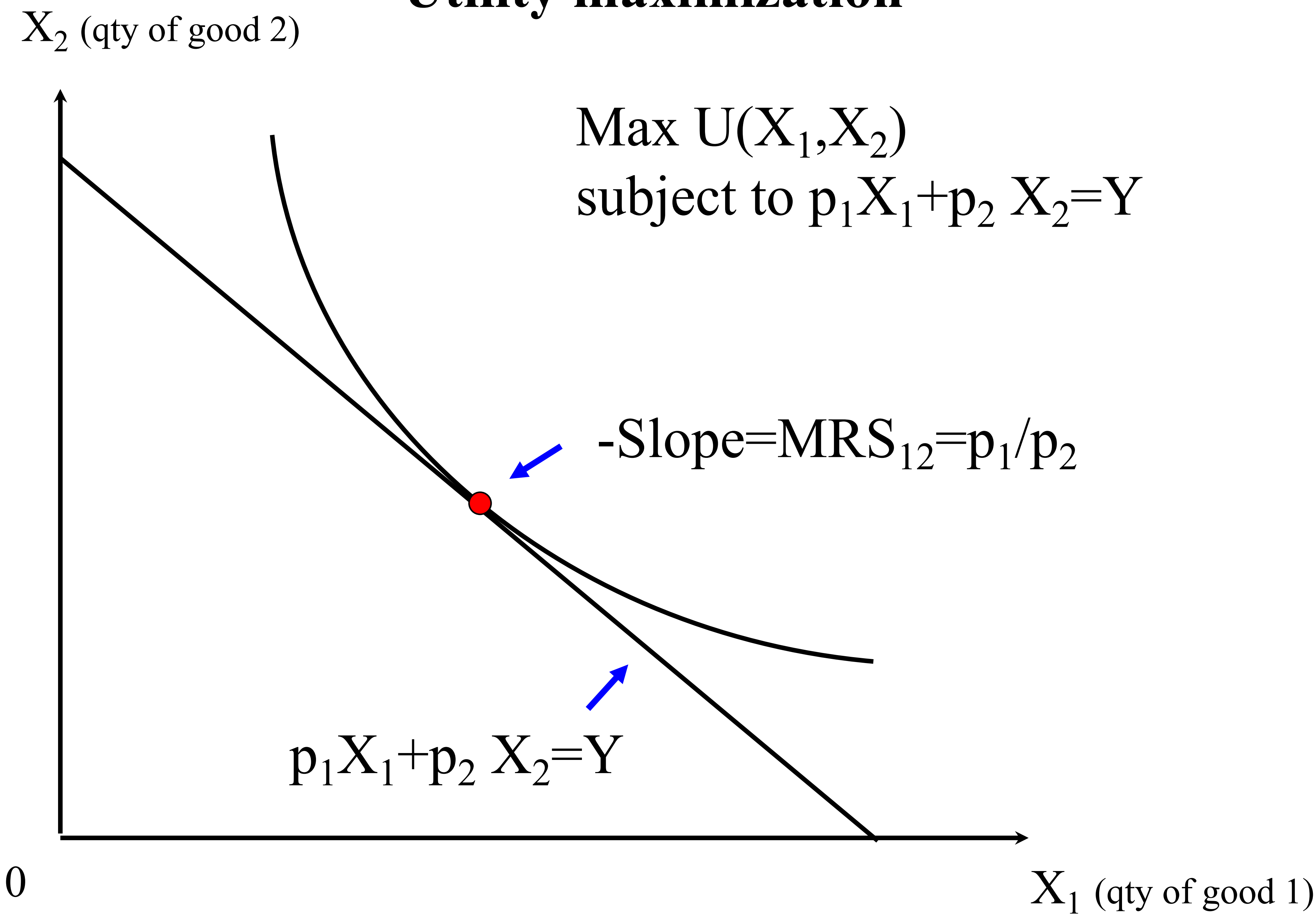


Utility maximization

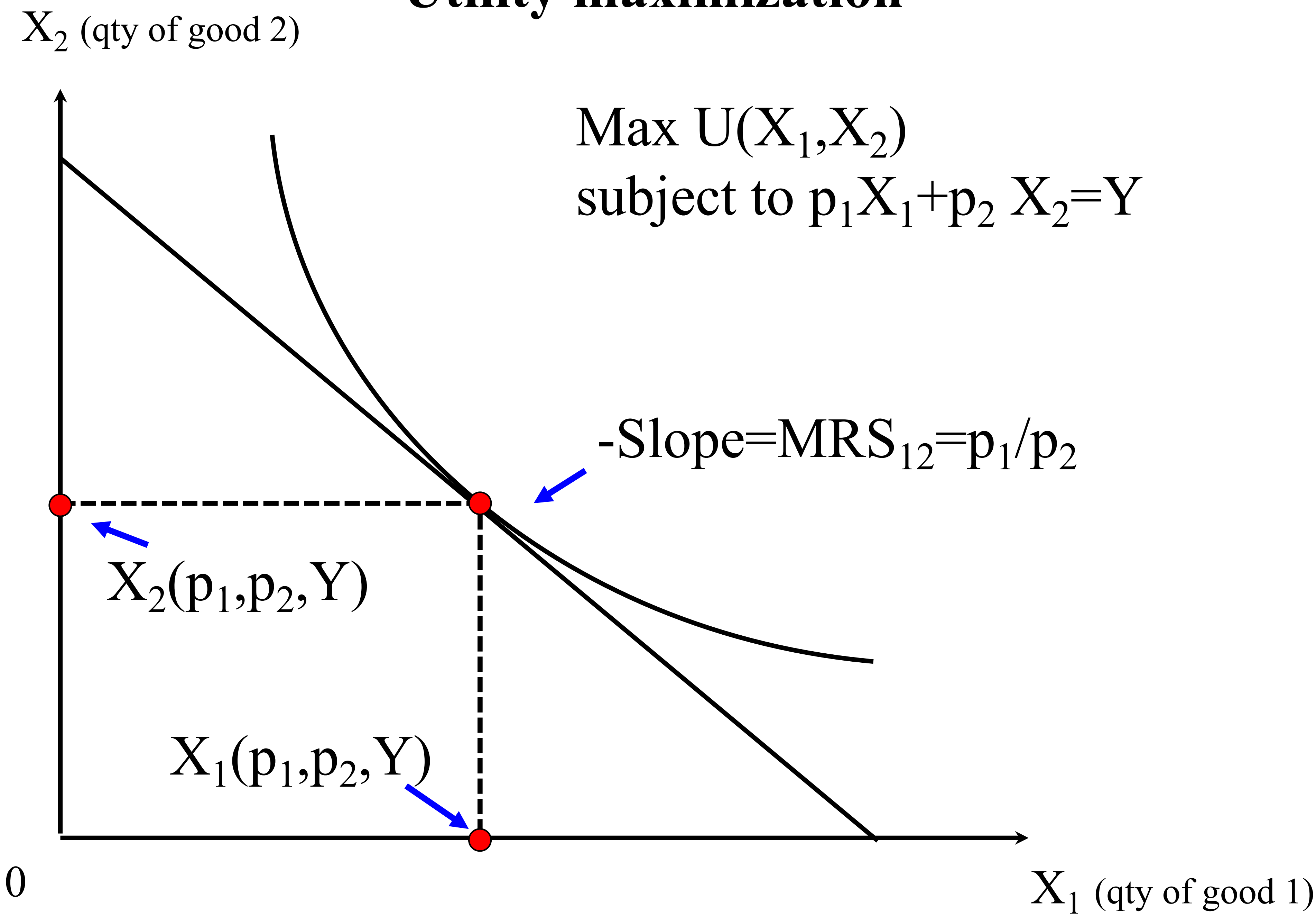
Max $U(X_1,X_2)$
subject to $p_1X_1+p_2 X_2=Y$



Utility maximization



Utility maximization



INCOME AND SUBSTITUTION EFFECTS

Let us denote by $p = (p_1, p_2)$ the price vector

Individual maximization generates demand functions $X_1(p, Y)$ and $X_2(p, Y)$

How does $X_1(p, Y)$ vary with p and Y?

Those are called price and income effects

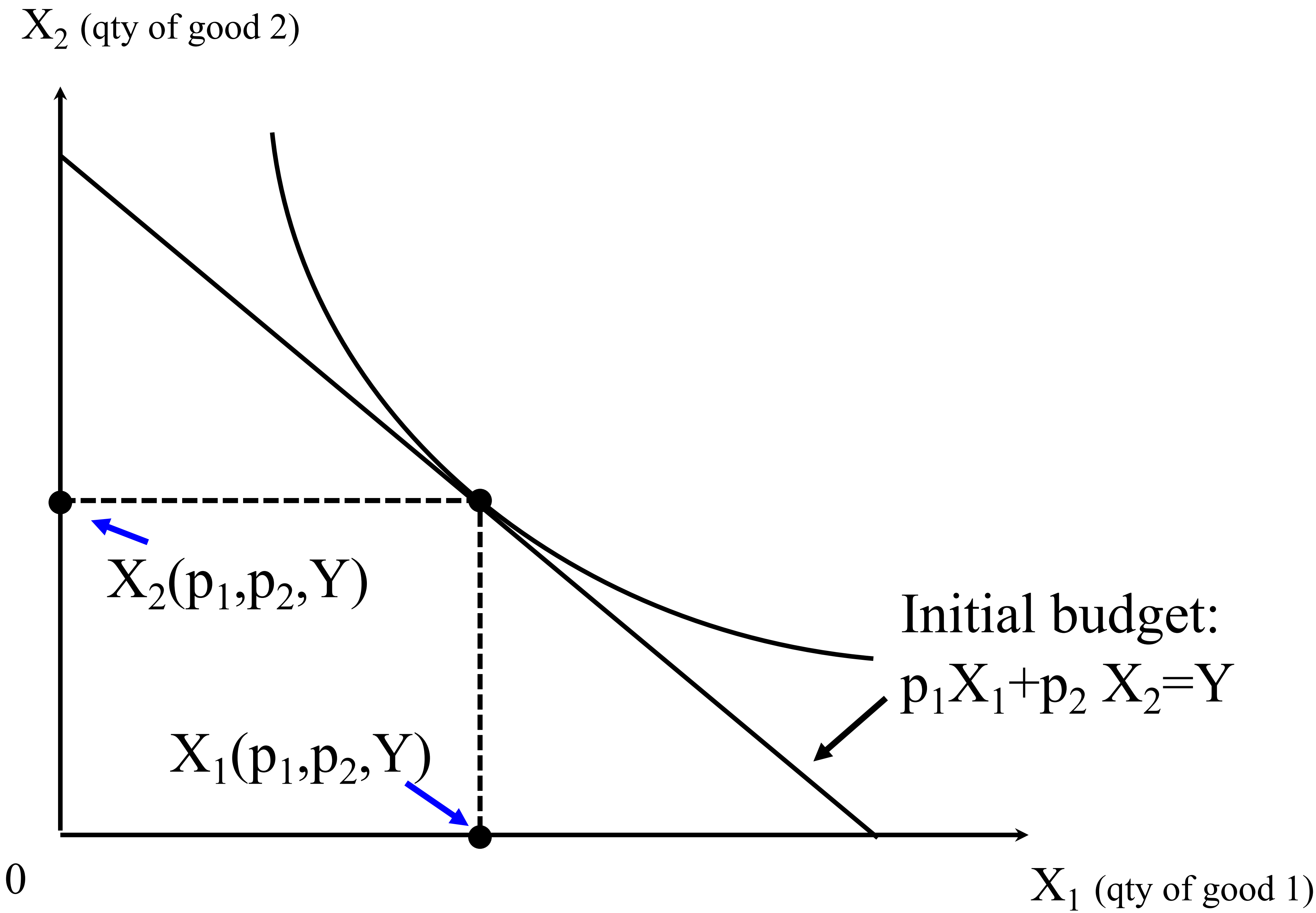
Example: $u(X_1, X_2) = \sqrt{X_1 \cdot X_2}$ then $MRS_{1,2} = X_2/X_1$.

Utility maximization implies $X_2/X_1 = p_1/p_2$ and hence $p_1X_1 = p_2X_2$

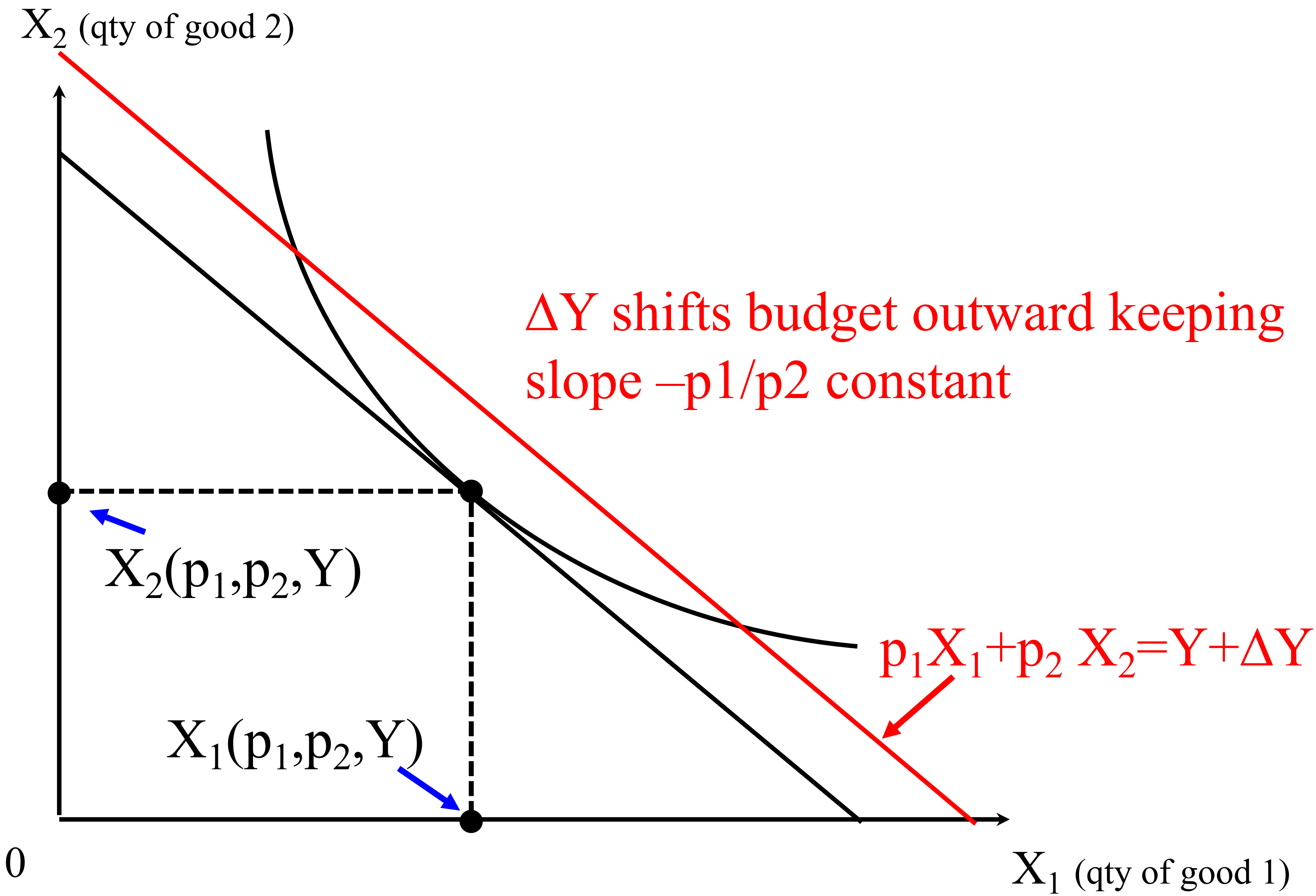
Budget constraint $p_1X_1 + p_2X_2 = Y$ implies $p_1X_1 = p_2X_2 = Y/2$

Demand functions: $X_1(p, Y) = Y/(2p_1)$ and $X_2(p, Y) = Y/(2p_2)$

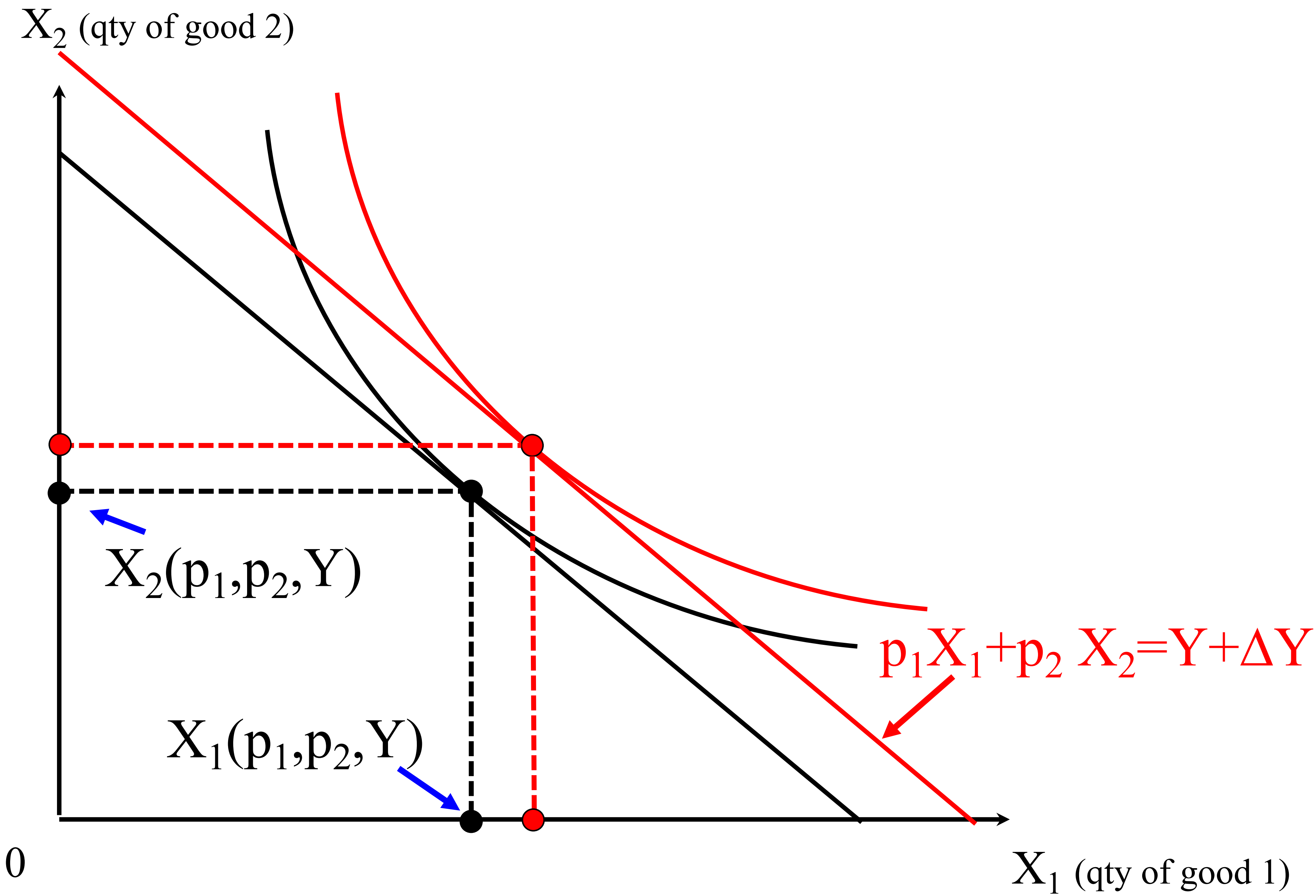
Income Effects: Y increases to Y+ΔY



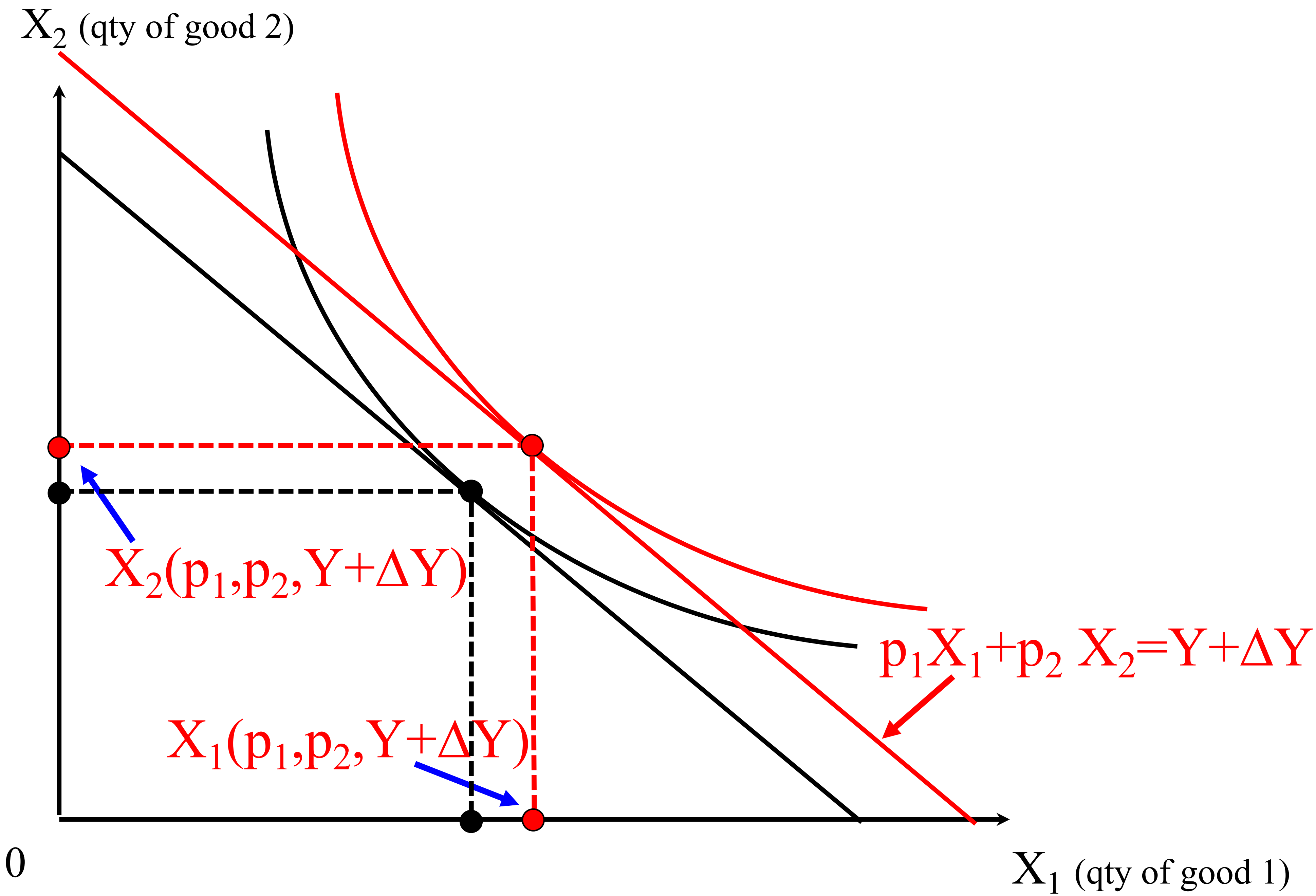
Income Effects: Y increases to Y+ΔY



Income Effects: Y increases to Y+ΔY



Income Effects: Y increases to Y+ΔY



INCOME EFFECTS

Income effect is the effect of giving extra income Y on the demand for goods: How does $X_1(p,Y)$ vary with Y?

Normal goods: Goods for which demand increases as income Y rises: $X_1(p,Y)$ increases with Y (most goods are normal)

Inferior goods: Goods for which demand falls as income Y rises: $X_1(p,Y)$ decreases with Y (example: you use public transportation less when you are rich enough to buy a car)

PRICE EFFECTS

How does $X_1(p_1, p_2, Y)$ vary with p_1?

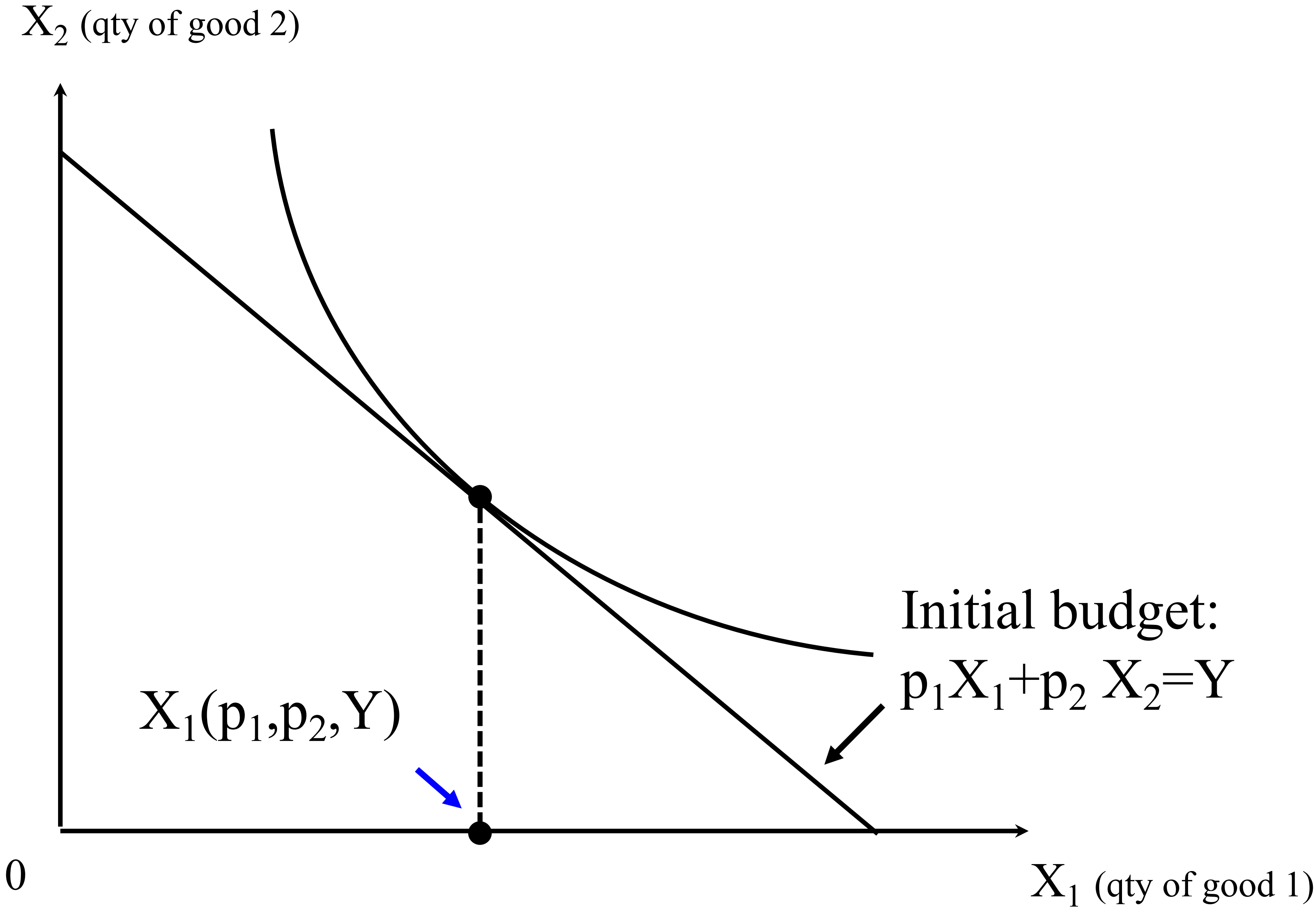
Changing p_1 affects the slope of the budget constraint and can be decomposed into 2 effects:

1) Substitution effect: Holding utility constant, a relative rise in the price of a good will always cause an individual to choose less of that good

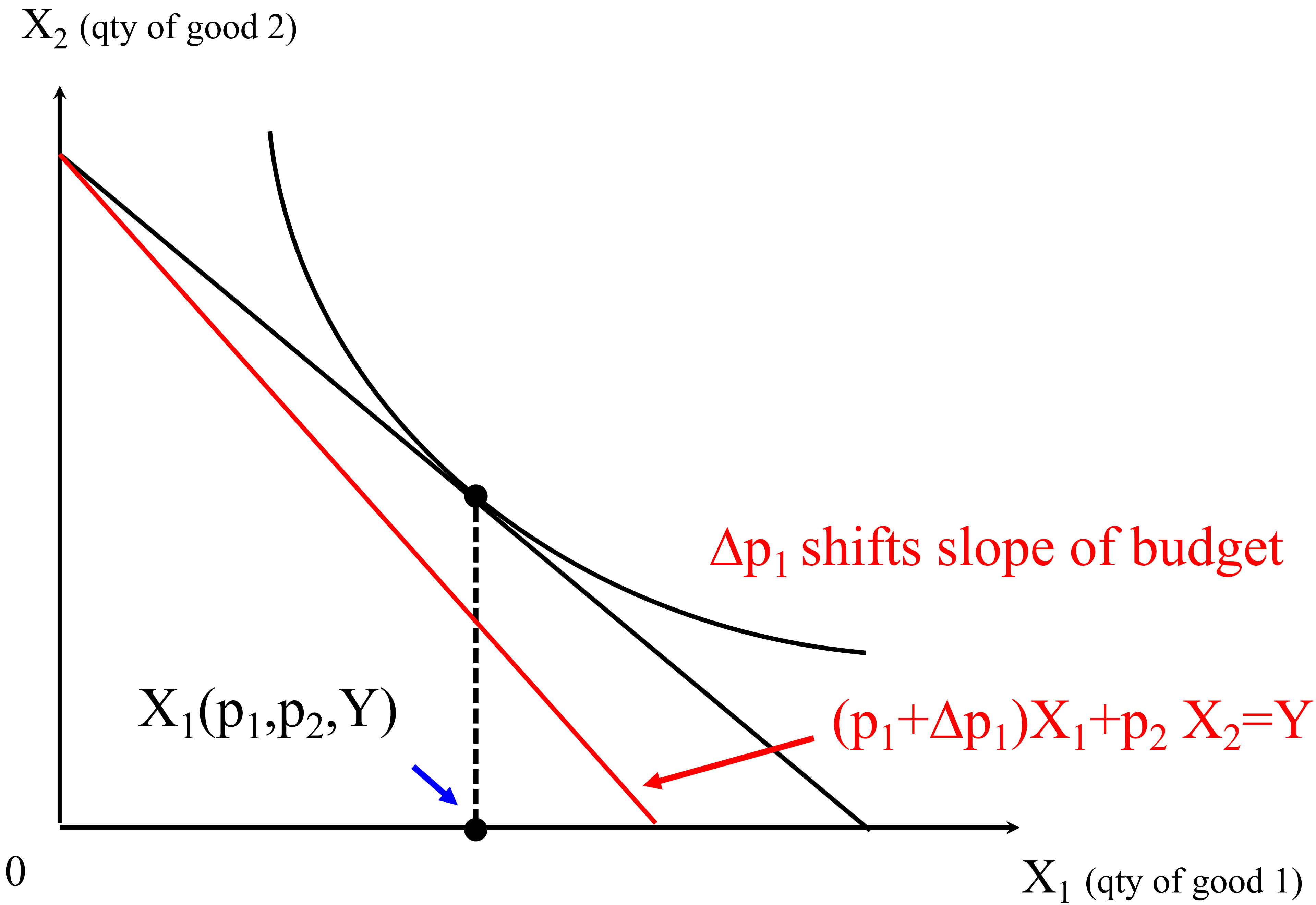
2) Income effect: A rise in the price of a good will typically cause an individual to choose less of all goods because her income can purchase less than before

For normal goods, an increase in p_1 reduces $X_1(p_1, p_2, Y)$ through both substitution and income effects

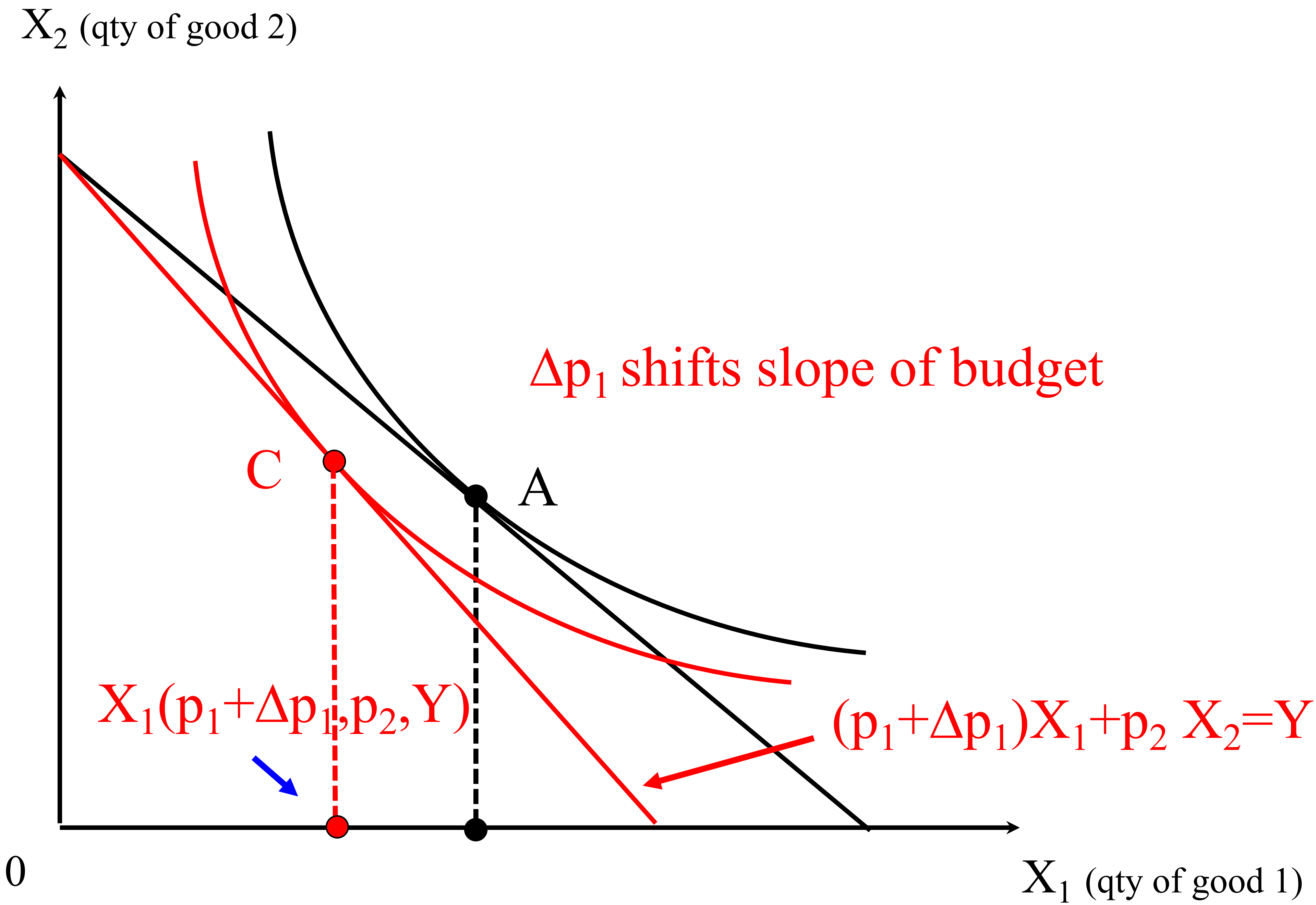
Price Effects: p_1 increases to $p_1+\Delta p_1$



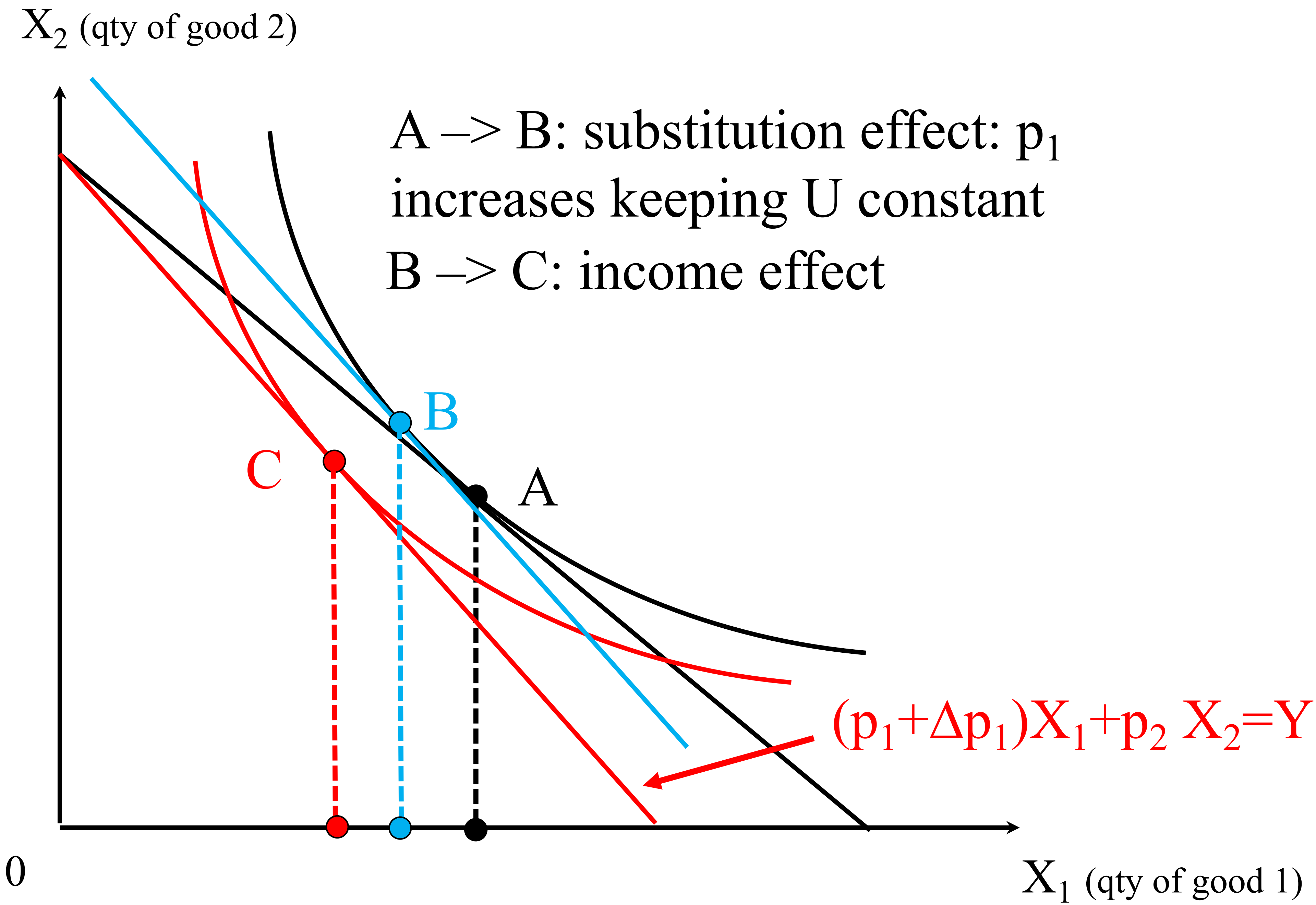
Price Effects: p_1 increases to $p_1+\Delta p_1$



Price Effects: p_1 increases to $p_1+\Delta p_1$



Price Effects: p_1 increases to $p_1+\Delta p_1$



AGGREGATE DEMAND

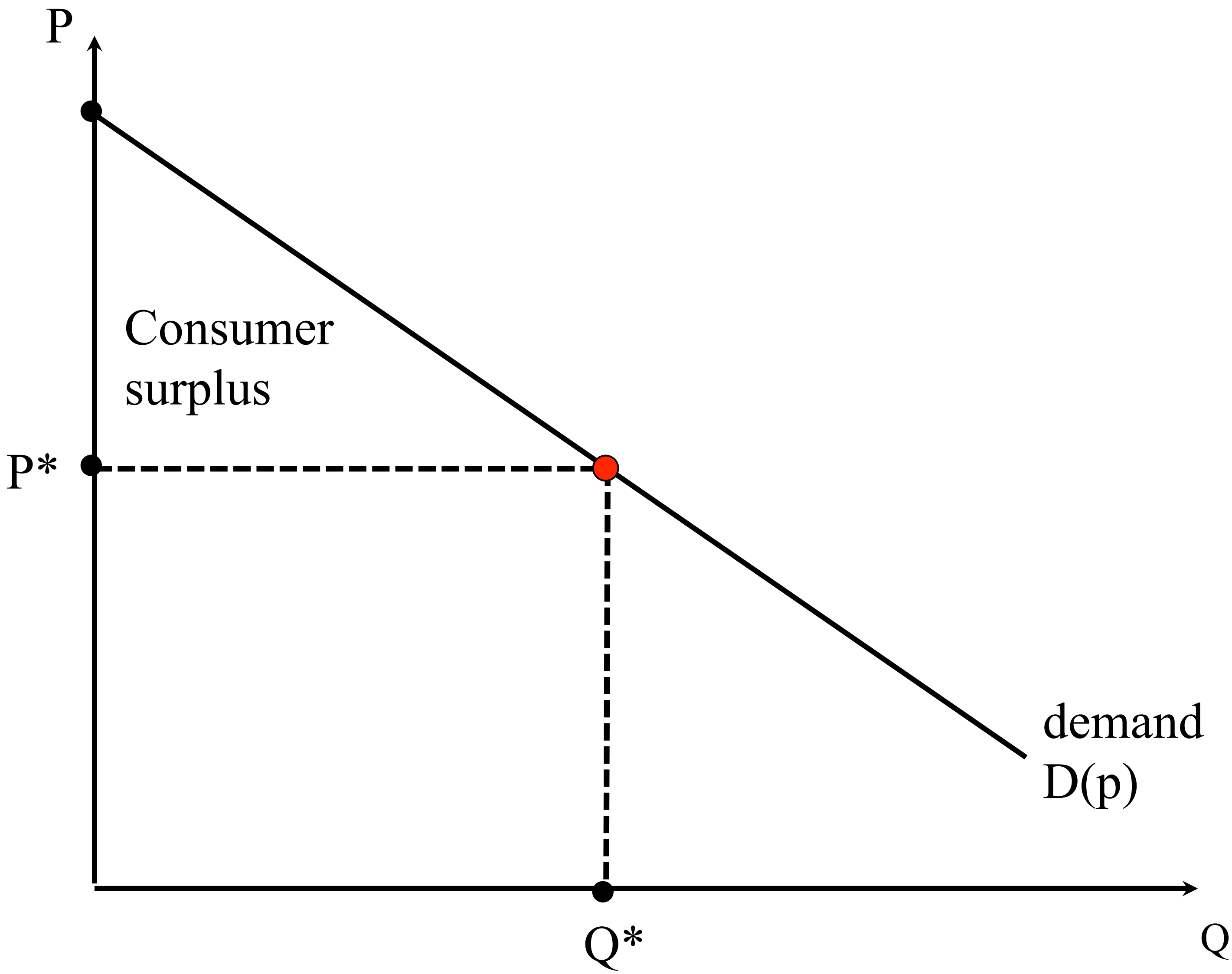
Each individual has a demand for each good that depends on the price p of the good. Aggregating across all individuals, we get aggregate demand $D(p)$ for the good

Basic rationalization: consumers maximize $v(Q) - p \cdot Q$ where $v(Q)$ is utility of consuming Q units (increasing and concave): First order condition $v'(Q) = p$ defines $Q = D(p)$.

At price p, demand is $D(p)$ and p is the $\$$ value for consumers of the marginal (last) unit consumed

First unit consumed generates utility $v'(0) = D^{-1}(0)$ and hence surplus $D^{-1}(0) - p$, last (marginal) unit consumed generates surplus $v'(Q) - p = 0$

$\Rightarrow$ Consumer surplus can be measured as area below the demand curve and above the price horizontal line


P
Consumer
surplus
P*
demand
D(p)
Q*
Q

ELASTICITY OF DEMAND

Elasticity of demand = The % change in demand caused by a 1% change in the price of that good:

$$\varepsilon^D = \frac{\text{\% change in quantity demanded}}{\text{\% change in price}} = \frac{\Delta D/D}{\Delta p/p} = \frac{p}{D}\frac{dD}{dp}$$

Elasticities are widely used because they are **unit free**

$\varepsilon^D = pD'(p)/D(p)$ is a function of p and hence can vary with p along the demand curve

When $D(p) = D_0 \cdot p^{\varepsilon}$ with D_0, ε fixed parameters, then $\varepsilon^D = \varepsilon$ is constant (called iso-elastic demand function)

PROPERTIES OF ELASTICITY OF DEMAND

1) Typically negative, since quantity demanded typically falls as price rises.

2) Typically not constant along a demand curve.

3) With vertical demand curve, demand is **perfectly inelastic** ($\varepsilon = 0$).

4) With horizontal demand curve, demand is **perfectly elastic** ($\varepsilon = -\infty$).

5) The effect of one good's prices on the demand for another good is the **cross-price** elasticity. Typically, not zero.

PRODUCERS

Producers (typically firms) use technology to transform inputs (labor and capital) into outputs (consumption goods)

Narrow economic view: Goal of producers is to maximize profits = sales of outputs minus costs of inputs

Production decisions (for given prices) define supply functions

Simple case: Profits $\Pi = p \cdot Q - c(Q)$ where $c(Q)$ is cost of producing quantity Q. $c(Q)$ is increasing and convex (means that $c'(Q)$ increases with Q).

Profit maximization: $\max_Q [p \cdot Q - c(Q)]$

$\Rightarrow c'(Q) = p$: marginal cost of production equals price

Defines the supply curve $Q = S(p)$.

SUPPLY CURVES

Supply curve $S(p)$ is the quantity that firms in aggregate are willing to supply at each price: typically upward sloping with price due to decreasing returns to scale

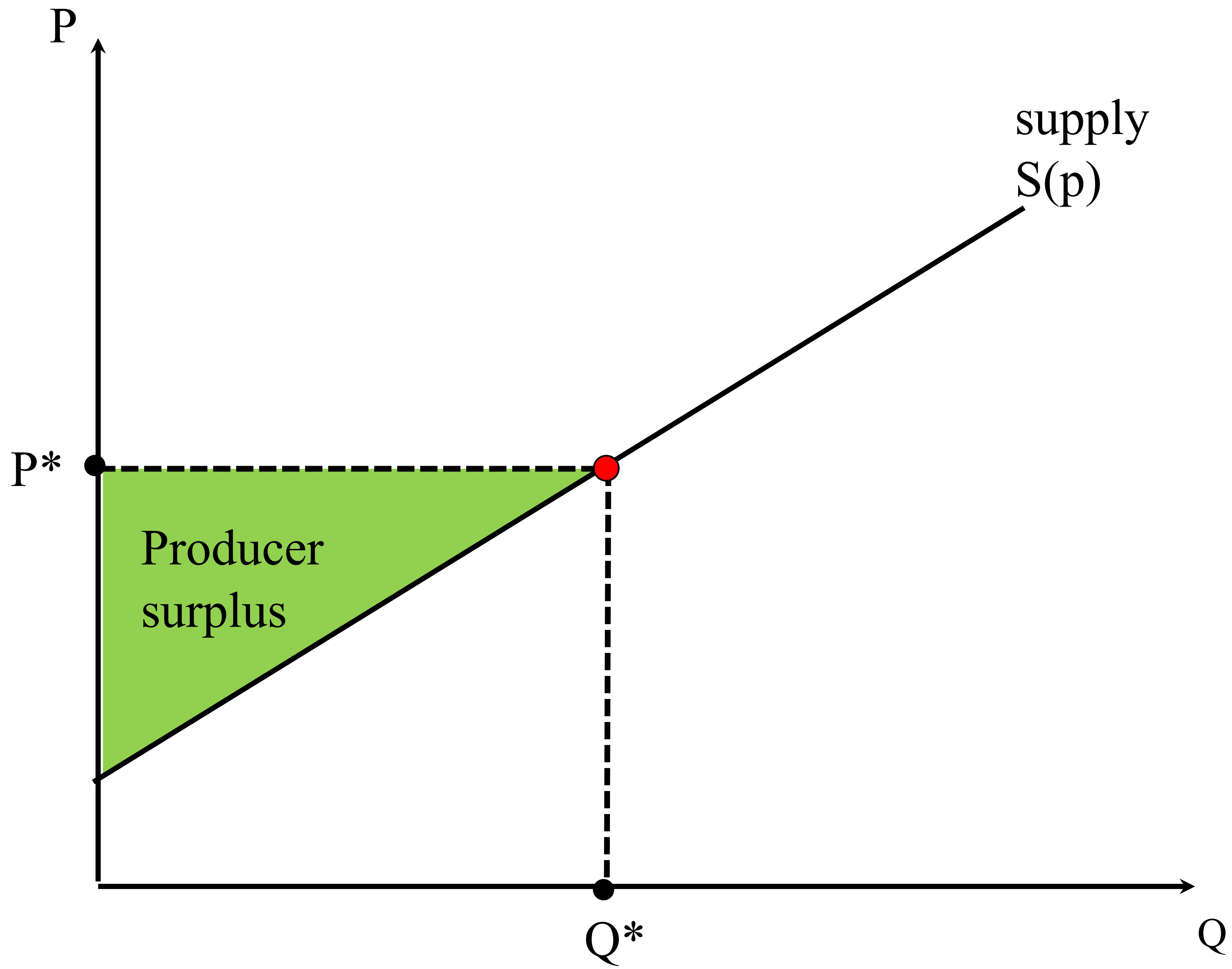
At price p, producers produce quantity $S(p)$, and the \$ cost of producing the marginal (last) unit is p

Elasticity of supply ε_S is defined as

$$\varepsilon_S = \frac{\text{\% change in quantity supplied}}{\text{\% change in price}} = \frac{\Delta S/S}{\Delta p/p} = \frac{p}{S}\frac{dS}{dp}$$

$\varepsilon^S = pS'(p)/S(p)$ is a function of p and hence can vary with p along the supply curve

When $S(p) = S_0 \cdot p^{\varepsilon}$ with S_0, ε fixed parameters, then $\varepsilon^S = \varepsilon$ is constant (called iso-elastic supply function)


P
supply
S(p)
P*
Producer
surplus
Q*
Q

MARKET EQUILIBRIUM

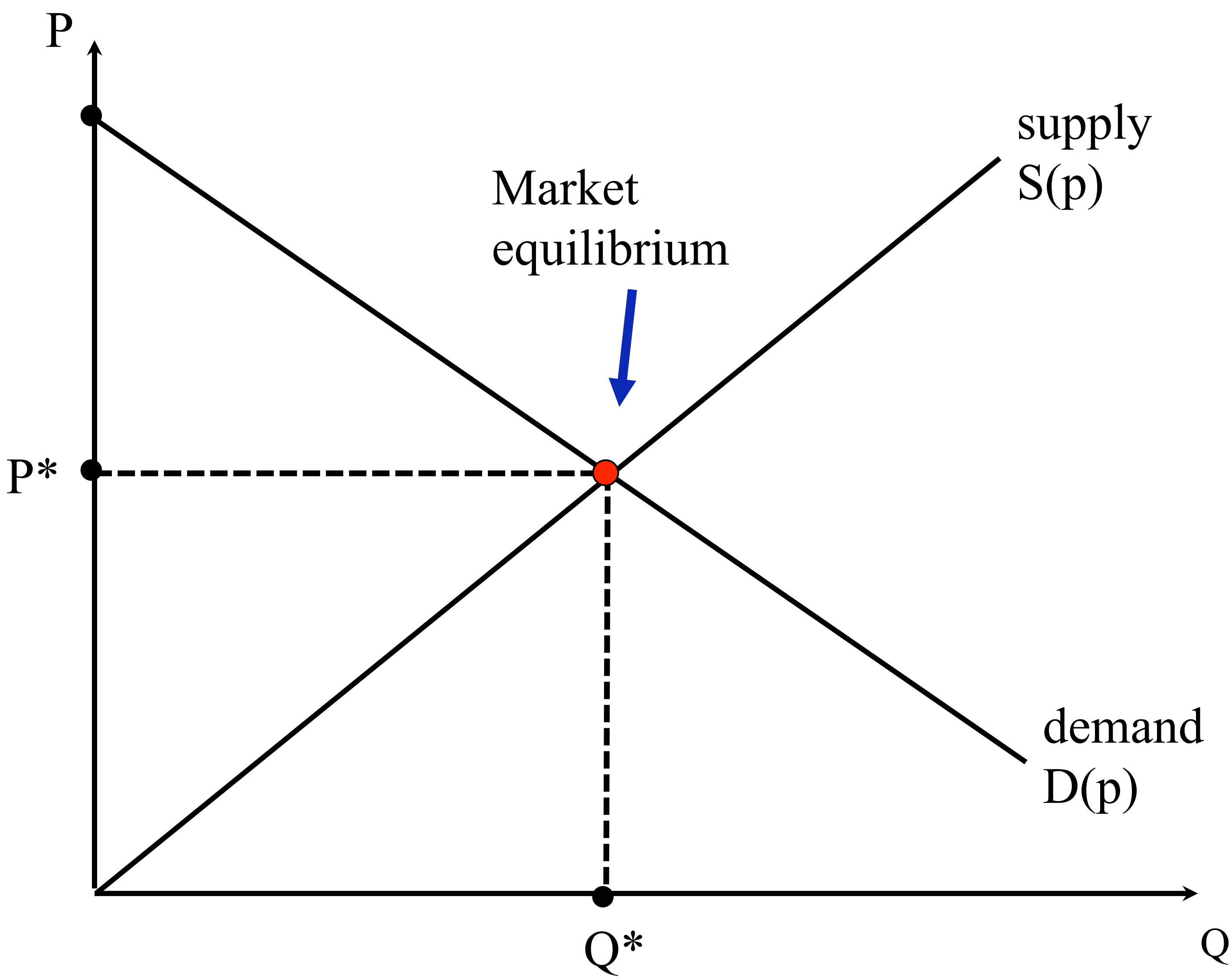
Consumers (demand side) and producers (supply side) interact on markets

Market equilibrium: The equilibrium is the price p^* such that $D(p^*) = S(p^*)$

In the simple diagram, p^* is unique if $D(p)$ decreases with p and $S(p)$ increases with p

If $p > p^*$, then supply exceeds demand, and price needs to fall to equilibrate supply and demand

If $p < p^*$, then demand exceeds supply, and price needs to increase to equilibrate supply and demand


P
supply
S(p)
Market
equilibrium
P*
demand
D(p)
Q*
Q

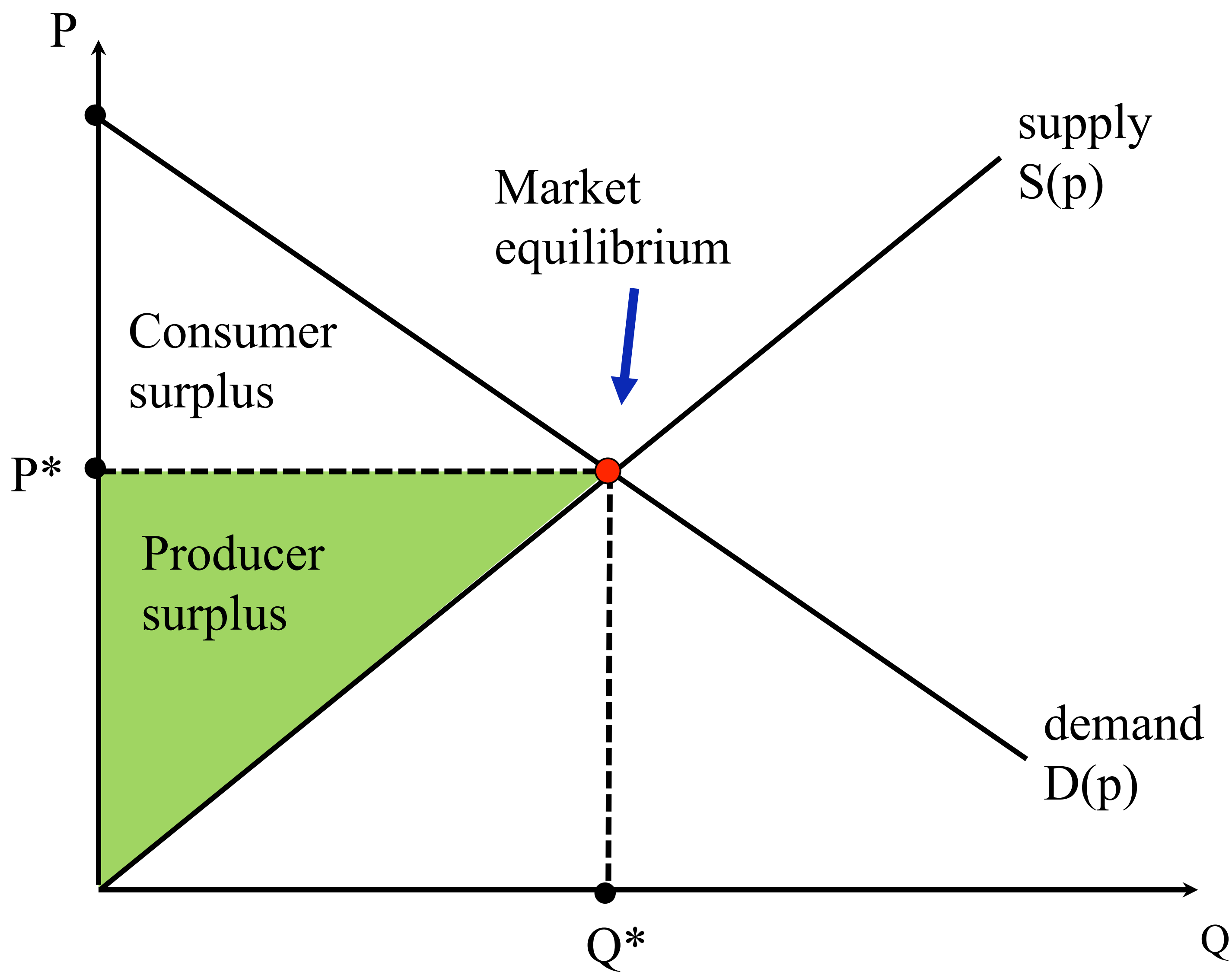
ECONOMIC SURPLUS

Economic surplus represents the net gains to society from all trades that are made in a particular market, and it consists of two components: consumer and producer surplus.

Consumer surplus: The benefit that consumers derive from consuming a good, above and beyond the price they paid for the good = area below demand curve and above market price

Producer surplus: The benefit producers derive from selling a good, above and beyond the cost of producing that good = area above supply curve and below market price

Total economic surplus: Consumer surplus + producer surplus = area above supply curve and below demand curve


P
Q
P*
Q*
Market
equilibrium
supply
S(p)
demand
D(p)
Consumer
surplus
Producer
surplus

Competitive Equilibrium Maximizes Economic Surplus

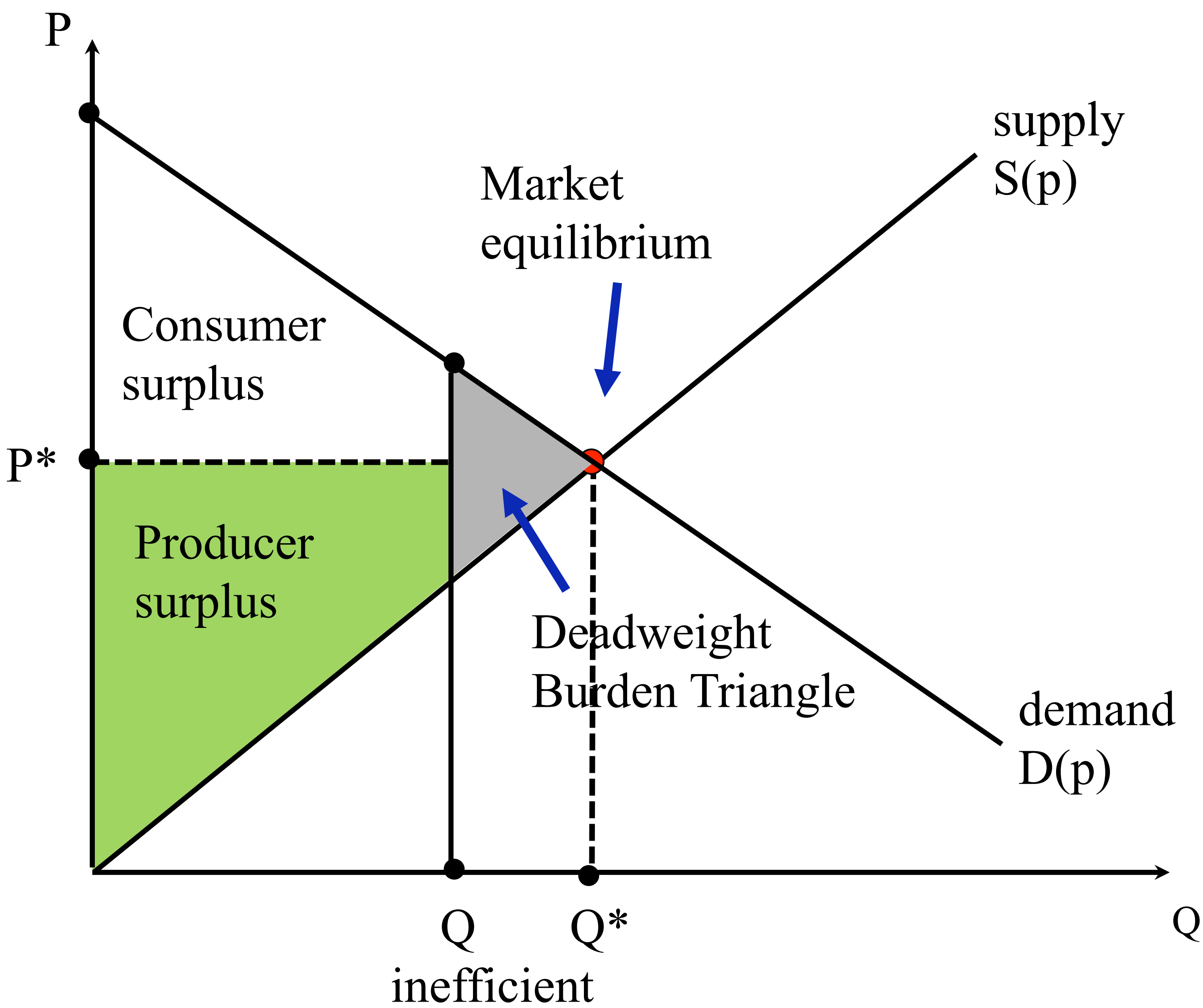
First Fundamental Theorem of Welfare Economics: The competitive equilibrium where supply equals demand, maximizes total economic surplus (sometimes called "efficiency")

Economic surplus just counts dollars regardless of who gets them (\$1 to rich producer better than \$.99 to poor consumer) $\Rightarrow$ 1st welfare theorem is blind to distributional aspects

Deadweight loss: The reduction in economic surplus from denying trades for which benefits exceed costs when quantity differs from the efficient quantity

Key rule: Deadweight loss triangle points to the efficient allocation, and grows outward from there

The simple efficiency result from the 1-good diagram can be generalized into the first welfare theorem (Arrow-Debreu, 1940s), most important result in economics


P
supply
S(p)
Market
equilibrium
Consumer
surplus
P*
Producer
surplus
Deadweight
Burden Triangle
demand
D(p)
Q
Q*
Q
inefficient

Generalization: 1st Welfare Theorem

1st Welfare Theorem: If (1) no externalities, (2) perfect competition [individuals and firms are price takers], (3) perfect information, (4) agents are rational, then private market equilibrium is **Pareto efficient**

Pareto efficient: Impossible to find a technologically feasible allocation that improves everybody's welfare

Pareto efficiency is desirable but a very weak requirement (a single person consuming everything is Pareto efficient)

Government intervention may be particularly desirable if the assumptions of the 1st welfare theorem fail, i.e., when there are **market failures** $\Rightarrow$ Govt intervention can potentially improve everybody's welfare

Second part of class considers such market failure situations

2nd Welfare Theorem

Even with no market failures, free market outcome might generate substantial inequality. Inequality is seen as one of the biggest issue with market economies.

2nd Welfare Theorem: Any Pareto Efficient allocation can be reached by

(1) Suitable redistribution of initial endowments [individualized **lump-sum** taxes based on individual characteristics and not behavior]

(2) Then letting markets work freely

$\Rightarrow$ No conflict between efficiency and equity

2nd Welfare Theorem Fallacy

In reality, 2nd welfare theorem does not work because redistribution of initial endowments is not feasible (because initial endowments cannot be observed by the government)

$\Rightarrow$ govt needs to use **distortionary** taxes and transfers based on economic outcomes (such as income or working situation)

$\Rightarrow$ Conflict between efficiency and equity: **Equity-Efficiency trade-off**

First part of class considers policies that trade-off equity and efficiency

Illustration of 2nd Welfare Theorem Fallacy

Suppose economy is populated 50% with disabled people unable to work (hence they earn \$0) and 50% with able people who can work and earn \$100

Free market outcome: disabled have \$0, able have \$100

2nd welfare theorem: govt is able to tell apart the disabled from the able [even if the able do not work]

⇒ can tax the able by \$50 [regardless of whether they work or not] to give \$50 to each disabled person ⇒ the able keep working [otherwise they'd have zero income and still have to pay \$50]

Real world: govt can't tell apart disabled from non working able

⇒ \$50 tax on workers + \$50 transfer on non workers destroys all incentives to work ⇒ govt can no longer do full redistribution ⇒ Trade-off between equity and size of the pie

SOCIAL WELFARE FUNCTIONS

Economists incorporate distributional aspects using **social welfare functions** (instead of just adding $ of economic surplus)

Social welfare function (SWF): A function that combines the utility functions of all individuals into an overall social utility function

General idea is that one dollar to a disadvantaged person might count more than one dollar to a rich person

UTILITARIAN SOCIAL WELFARE FUNCTION

With a utilitarian social welfare function, society's goal is to maximize the sum of individual utilities:

$$SWF = U_1 + U_2 + ... + U_N$$

The utilities of all individuals are given equal weight, and summed to get total social welfare

If marginal utility of money decreases with income (satiation), utilitarian criterion values redistribution from rich to poor

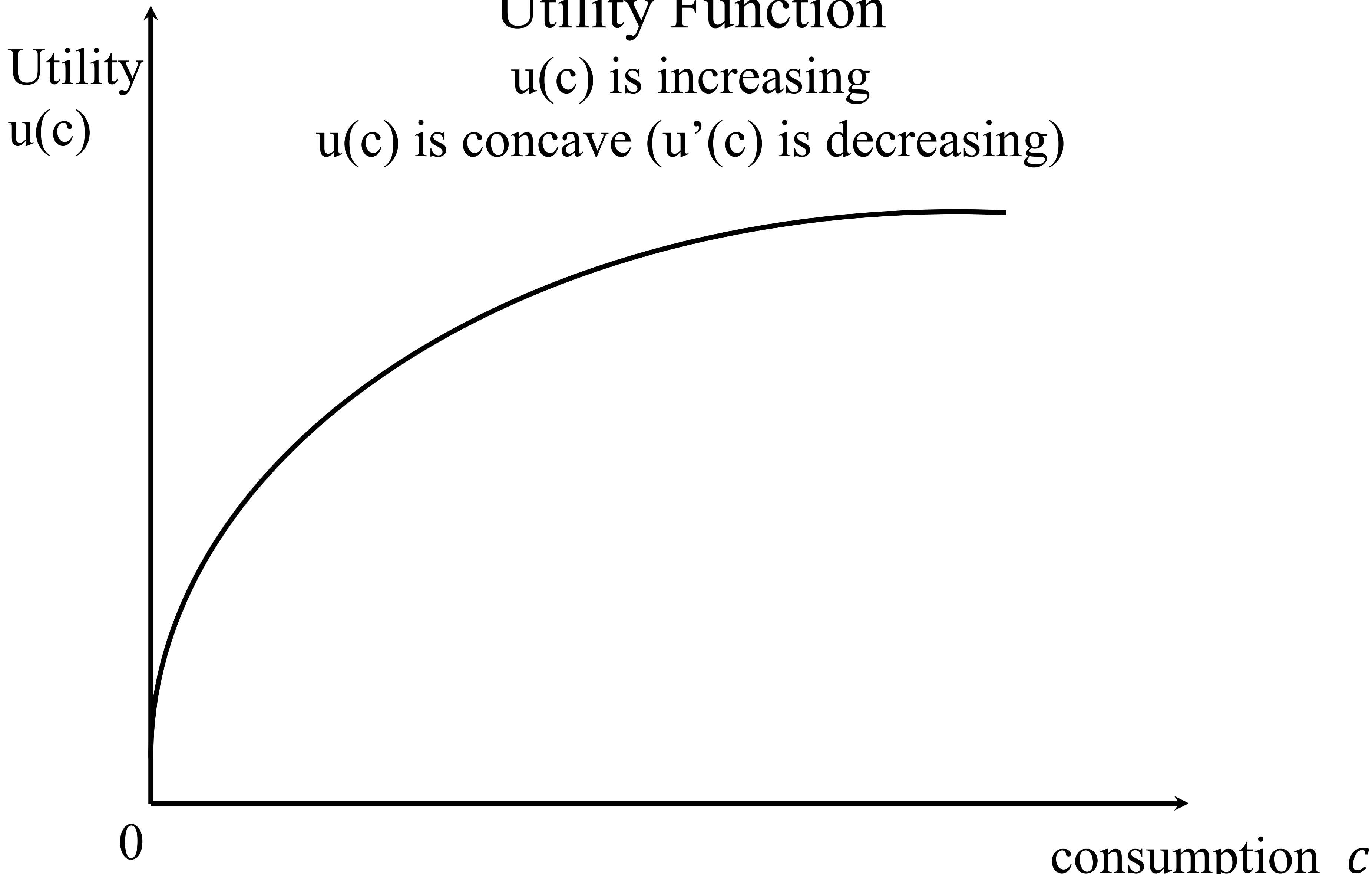
Taking \$1 for a rich person decreases his utility by a small amount, giving the \$1 to a poor person increases his utility by a large amount

$\Rightarrow$ Transfers from rich to poor increase total utility

Utility Function

u(c) is increasing

u(c) is concave (u'(c) is decreasing)



RAWLSIAN SOCIAL WELFARE FUNCTION

Rawls (1971) proposed that society's goal should be to maximize the well-being of its worst-off member. The Rawlsian SWF has the form:

$$SWF = \min(U_1, U_2, ..., U_N)$$

Since social welfare is determined by the minimum utility in society, social welfare is maximized by maximizing the well-being of the worst-off person in society (=maxi-min)

Rawlsian criterion is even more redistributive than utilitarian criterion: society wants to extract as much tax revenue as possible from the middle and rich to make transfers to the poor as large as possible

OTHER SOCIAL JUSTICE PRINCIPLES

Standard welfarist approach is based on individual utilities. This fails to capture important elements of actual debates on redistribution and fairness

1) Just deserts: Individuals should receive compensation congruent with their contributions (libertarian).

$\Rightarrow$ Taxes should be tailored to government benefits received

2) Commodity egalitarianism: Society should ensure that individuals meet a set of basic needs (seen as rights)

$\Rightarrow$ Rich countries today consider free education, universal health care, retirement/disability benefits, free sanitary products for women as rights

3) Equality of opportunity: Society should ensure that all individuals have equal opportunities for success

$\Rightarrow$ Individuals should be compensated for inequalities they are not responsible for (e.g., family background, inheritance, intrinsic ability) but not for inequalities they are responsible for (being hard working vs. loving leisure)

TESTING PEOPLE SOCIAL PREFERENCES

Saez-Stantcheva '16 survey people online (using Amazon MTurk) by asking hypothetical questions to elicit social preferences. Key findings:

1) People typically do not have "utilitarian" social justice principles (consumption lover not seen as more deserving than frugal person)

2) People put weight on whether income has been earned through effort vs. not (hard working vs. leisure lover)

3) People put a lot of weight of what people would have done absent the government intervention (deserving poor vs. free loaders)

ACTUAL SOCIAL PREFERENCES

General conclusion: People favor redistribution if they feel inequalities are "unfair" but views on what is fair differ

$\Rightarrow$ Redistribution supported when people don't have control [education for children, health insurance for the sick, retirement/disability benefits for the elderly/disabled unable to work]

$\Rightarrow$ Less support when people have some or full control [unemployment, being low income]

$\Rightarrow$ Less support when people don't "belong" (us vs. them)

Conservatives tend to frame things: individuals have control (personal responsibility), govt should just enforce rules

Liberals tend to frame things: many forces in society beyond individuals' control, society should provide nurturing
See Lakoff (1996) for how liberals and conservative think

Conclusion: Two General Rules for Govt Intervention

1) Market Failures: Government intervention can help if there are market failures

2) Redistribution: Free market generates inequality. Govt taxes and spending can reduce inequality

Most of this module will focus on 2), last part of course will analyze 1)

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